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COMPARTMENT VENTING AND PIPE FLOW WITH HEAT ADDITION

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NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER

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ABSTRACT

One-dimensional quasi-steady theory is used to develop an engineering method to determine the time-dependent pressure in vented launch-vehicle compartments during the first few minutes of the ascending flight.

This method, by nature an iteration procedure, is intended to provide the structural design engineer with the reduced loads on the compartment walls resulting from the venting process. The basic program is set up for the CDC-3200 digital computer which can handle presently only up to N = 5 compartments where the inner compartments must have only one connecting orifice and the last compartment can have up to NV = 5 orifices venting into the atmosphere. Furthermore, the compartments have to be placed in series. Though the compartment and orifice number can be raised indefinitely, it is advisable to restrict the number to as few as possible to keep the computation time low.

The basic program has been extended to offer combinations of compartment and connecting orifices. Compartment leaks and their accompanying coefficients, as well as venting through a duct of varying cross section, have been included. The effect of heating or cooling the duct flow can also be computed.

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DEFINITION OF SYMBOLS

Symbol	Definition	Dimension
A	orifice area	m^2
a	mean radius of earth	m^2
C	radial clearance	m
c	velocity of sound	m sec^{-1}
H	geopotential altitude	m
L	flow length	m
R_H, D_H	hydraulic radius, diameter	m
V	volume	m^3
X	length of tube	m
Z	geodetic altitude	m
g_o	gravity	m sec^{-2}
M	Mach number	-
t	time	sec
v	velocity of gas	m sec^{-1}
c_p	specific heat at constant pressure	m Kg Kg^{-1} $°K^{-1}$
c_v	specific heat at constant volume	m Kg Kg^{-1} $°K^{-1}$
f	friction factor	-
h	enthalpy	m Kg
K, K_o	loss coefficient, contraction coefficient	-
L'	temperature gradient	°K Km^{-1}
m	mass	Kg sec^2 m^{-1}
$\dot{m}$	mass flow rate	Kg sec m^{-1}

DEFINITION OF SYMBOLS

Symbol	Definition	Dimension
M_*	molecular weight	Kmol Kg^{-1}
$P, \bar{P}$	pressure, mean pressure	Kg m^{-2}
Q	heat per unit mass of gas entering the volume	Kg m^2 Kg^{-1} sec^{-1}
R_e	Reynolds number	-
R	gas constant	m Kg Kg^{-1} $°K^{-1}$
T	temperature	°K
W_x	external work	Kg m
γ	specific heat ratio	-
ρ	density	Kg sec^2 m^{-4}
μ	absolute viscosity	Kg m^{-1} sec^{-1}
τ_x	shearing stress	Kg m^{-2}
ζ	loss coefficient for pipes other than friction	
$\emptyset$	area ratio A_0/A_1	

Subscripts

1,2,i	designates sections under investigation
t	total flow properties, total pressure, total density

Throughout this report the technical system of units is used; therefore the unit Kg is the Kg force = Kp (Kilopond).

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COMPARTMENT VENTING AND PIPE FLOW WITH HEAT ADDITION

SUMMARY

One-dimensional quasi-steady theory is used to develop an engineering method to determine the time-dependent pressure in vented launch-vehicle compartments during the first few minutes of the ascending flight.

This method, by nature an iteration procedure, is intended to provide the structural design engineer with the reduced loads on the compartment walls resulting from the venting process. The basic program is set up for the CDC-3200 digital computer which can handle presently only to N = 5 compartments where the inner compartments must have only one connecting orifice and the last compartment can have up to NV = 5 orifices venting into the atmosphere. Furthermore, the compartments have to be placed in series. Though the compartment and orifice number can be raised indefinitely, it is advisable to restrict the number to as few as possible to keep the computation time low.

The basic program has been extended to offer combinations of compartment and connecting orifices. Compartment leaks and their accompanying coefficients, as well as venting through a duct of varying cross section, have been included. The effect of heating or cooling the duct flow can also be computed.

I. INTRODUCTION

Any internal launch-vehicle space bounded by rigid walls is defined as a compartment. These compartments must be depressurized or vented during the ascending portion of the flight trajectory when the surrounding atmospheric pressure decreases rather rapidly. Depressurization is generally obtained by orifices cut into the rigid wall, or openings leading into another compartment which is vented to the surrounding atmosphere. The size and shape of the hole, as well as the pressure differential from one chamber to another or to the atmosphere, determine the mass flux out of the compartment. Because of the mass transfer to the next lower compartment, the density in the first compartment will decrease and so will the internal pressure. Any inflowing mass from upstream compartments, however, will increase the pressure again. This indicates the dependence of the gas properties of the compartment on downstream and upstream conditions. However, should choking occur in one of the orifices or in the vent duct, the line of dependence is then restricted to the downstream or upstream portion of the flow only. For the isotropical orifice flow, empirical correction factors are used which are a combination of contraction coefficients (vena contracta), friction coefficients, and, for the outside vents, the influence of the outside tangential flow.

Occasionally compartments are vented through ducts which might consist of pipes with varying diameters. Adiabatic flow theory is then applied with suitable pipe friction factors to calculate flow properties in the duct. Heat addition to the flow is neglected when the duct is not too long, since the time range involved is relatively short. However, for long pipes heat addition to the flow can have a definite effect, especially when the temperature difference between ambient and pipe flow is large.

The equations for the atmospheric data are taken from the U. S. Standard Atmosphere, 1962 [9] with a simplified calculation of the geopotential altitude. These data can be corrected for a specific launch location if a table of correction factors for the pressure is added as a subroutine to the program.

II. THE ANALYTICAL APPROACH

A. Simple Orifice Flow

The venting problem, in reality a time-dependent unsteady problem, is simplified to a quasi-steady one-dimensional problem; all properties are therefore uniform over each cross section. Further assumptions are that the heat transfer between the surrounding compartment walls and the gas within is negligible, and the gas remains perfect throughout the entire venting process.

The approach consists of applying the mass and energy balance equation and the equation of state. We begin with the mass conservation law, expressed in general as

$$\frac{\partial (m)_{cv}}{\partial t} = \sum \dot{m}_i - \sum \dot{m}_o \qquad (1)$$

which states that the rate of accumulation of mass within the control volume (c.v.) is equal to the excess of the incoming rate of flow

$$\sum \dot{m}_i$$

over the outgoing rate of flow

$$\sum \dot{m}_o.$$

At any instance of flow,

$$(m)_{cv} = \int_m dm_{cv} = \int_{cv} \rho dV, \tag{2}$$

where dV is an element of the control volume, ρ is the local mass density of the control volume, and the integral is to be taken over the entire control volume. Furthermore,

$$\frac{\partial (m)_{cv}}{\partial t} = \int_{cv} \frac{\partial \rho}{\partial t} dV \tag{3}$$

and

$$\sum \dot{m}_i = \int \rho v_n \, dA_i; \quad \sum \dot{m}_o = \int \rho v_n \, dA_o. \tag{4}$$

Equation (1) can now be written as

$$\int_{cv} \frac{\partial \rho}{\partial t} dV = \int \rho v_n \, dA_i - \int \rho v_n \, dA_o, \tag{5}$$

where v_n is the velocity component normal to dA. The control volume is now supposed to be the compartment, and with the assumption of one-dimensional flow we obtain

$$\frac{d\rho}{dt} V = \sum_n \dot{m}_i - \sum_m \dot{m}_o. \tag{6}$$

For steady flow, $d\rho/dt = 0$, and the incoming mass is equal to the outgoing mass. In equation (6), n and m designate the openings in the compartment through which gas can flow into and out of the chamber, respectively. For the venting problem, we allow the density ρ to change with time according to the difference of mass flux rate.

The energy equation of steady flow relates the external work effect and the external heat exchange to the increase in the flux enthalpy, kinetic energy, and potential energy passing through the control surface; then

$$m(dQ - dW_x) = m(h + dh) - mh + m\left(\frac{v^2}{2} + d\,\frac{v^2}{2}\right) - m\,\frac{v^2}{2} + m(z + dz) - mz, \tag{7}$$

where dQ is the net heat added to the stream from sources external to the main stream per unit mass of gas entering the control surface. Likewise, dW_x is the external work delivered to the outside body per unit mass of gas entering the control boundary. For our problem, $dW_x = 0$, and z, the height of the stream centerline to datum line, is negligible for gas flow. Then equation (7) becomes

$$mdQ = mdh + md(v^2/2). \tag{8}$$

The enthalpy h is a function of the temperature only; therefore,

$$dh = c_p dT, \tag{9}$$

and equation (8) becomes

$$mdQ = mc_p dT + md(v^2/2). \tag{10}$$

According to our assumption that no heat is added from the outside, $dQ = 0$, and equation (10) becomes

$$dh = -d(v^2/2). \tag{11}$$

The mass flow rate is now given by the continuity equation as

$$\dot{m} = \rho vA. \tag{12}$$

With the equation of state

$$P = \rho RT, \tag{13}$$

and

$$c_p T = \frac{\gamma}{\gamma - 1} \frac{P}{\rho} = \frac{1}{\gamma - 1} c^2,$$

the velocity can be obtained from the energy equation for adiabatic flow,

$$v^2 + 2c_p T = \text{const.}$$

By assuming that the velocity in the upstream compartment is negligibly small, $v_1 = 0$, we obtain finally for the velocity with which mass is flowing out of the compartment (Figure 1)

$$v_{ex} = \sqrt{\frac{2\gamma}{\gamma - 1} \left[\frac{P_1}{\rho_1} - \frac{P_{ex}}{\rho_{ex}}\right]}. \tag{14}$$

With the relation between pressure and density for an isentropic process of a perfect gas between two stations,

$$\frac{\rho_{ex}}{\rho_1} = (P_{ex}/P_1)^{1/\gamma}, \tag{15}$$

substitution of equations (13) through (15) into (12) yields the mass out of the compartment:

$$\dot{m} = AK\sqrt{2P_1\rho_1}\left(\frac{P_{ex}}{P_1}\right)^{1/\gamma}\sqrt{\frac{\gamma}{\gamma - 1}\left[1 - \left(\frac{P_{ex}}{P_1}\right)^{\frac{\gamma-1}{\gamma}}\right]}\,. \tag{16}$$

The discharge from a compartment orifice is considered to be an isentropic process, since we used isentropic relations for establishing the mass flow rate. Any losses of mass flow rate due to total pressure and contraction are represented by the loss coefficient $K < 1$. The pressures P_{ex} and P_1, as well as the density ρ_1, have to be representative mean values for a time-dependent process.

If the flow in the orifice becomes sonic, the mass flow is independent of the pressure difference across the orifice:

$$\dot{m} = AK\sqrt{2P_1\rho_1}\left(\frac{2}{\gamma + 1}\right)^{\frac{1}{\gamma-1}}\sqrt{\frac{\gamma}{\gamma + 1}}\,. \tag{17}$$

Because of the outflowing and inflowing mass, the density, pressure, and other properties within the compartment are changed. To obtain the necessary relation from equation (6), we approximate the differential quotient $d\rho/dt$ by a difference quotient:

$$\frac{d\rho}{dt} \approx \frac{\Delta\rho}{\Delta t} = \frac{P(t + \Delta t) - P(t)}{\Delta t} = \frac{\rho(t_1) - \rho(t_o)}{t_1 - t_o}\,. \tag{18}$$

Substituting equation (18) into equation (6) results in the following expression for the density:

$$\rho(t_1) = \rho(t_o) \left[1 + \frac{\Delta t \left[\sum \dot{m}_i - \sum \dot{m}_o \right]}{\rho(t_o)\ V} \right], \tag{19}$$

and according to the gas relation (equation (15)) the pressure at time t_1 becomes

$$P(t_1) = P(t_o) \left[1 + \frac{\Delta t \left[\sum \dot{m}_i - \sum \dot{m}_o \right]}{\rho(t_o)\ V} \right]^{\gamma}. \tag{20}$$

The pressure, density, and mass flow terms in equations (20) and (16) have to agree with each other within the compartment during a particular time step, $\Delta t = (t_1 - t_o)$. Since equation (16), the mass flow rate equation, uses mean values of the pressures, an iteration procedure is necessary so that P_1 represents a mean value of $P(t_1)$ and $P(t_o)$ in the particular time interval Δt.

To circumvent at least one iteration procedure for finding the mean pressure value of a single compartment, a series approximation of equation (16) and (20) may be used provided that the pressure change for a small time step can be assumed to be linear.

If we set

$$P_1 = \bar{P}_1 = \frac{P_1(t_1) + P_1(t_o)}{2},$$

and if

$$\frac{\bar{P}_{ex}}{\bar{P}_1}$$

(where $\bar{\bar{P}}_{ex}$ is also a mean value of the external pressure at that particular time step) does not differ very much from unity, a series approximation can be made with

$$X = \left(1 - \frac{\bar{\bar{P}}_1}{P_1(t_o)}\right) \ll 1. \qquad (21)$$

Departing from equation (16) with the above notation, we obtain for n vent holes

$$\sum_n \dot{m}_o = \sum_n A_n K_n \sqrt{2\bar{\bar{P}}_1\bar{\bar{\rho}}_1} \left(\frac{\bar{\bar{P}}_{ex_n}}{\bar{\bar{P}}_1}\right)^{1/\gamma} \sqrt{\frac{\gamma}{\gamma - 1}} \left[1 - \left(\frac{\bar{\bar{P}}_{ex_n}}{\bar{\bar{P}}_1}\right)^{\frac{\gamma-1}{\gamma}}\right]^{1/2}. \qquad (22)$$

With the isentropic gas relation, we can introduce the pressure of the compartment at time (t_o), $P(t_o)$, which is assumed to be known since it is the pressure at t_1 of the previous time step.

$$\frac{\bar{\bar{P}}_{ex}}{\bar{\bar{P}}_1} = \frac{\bar{\bar{P}}_{ex}}{P_1(t_o)} \cdot \frac{P_1(t_o)}{\bar{\bar{P}}_1} \qquad (23)$$

and the square root term is

$$\sqrt{2\bar{\bar{P}}_1\bar{\bar{\rho}}_1} = \sqrt{2P_1(t_o)\ \rho_1(t_o)} \left(\frac{\bar{\bar{P}}_1}{P_1(t_o)}\right)^{\frac{\gamma+1}{2\gamma}}. \qquad (24)$$

With the abbreviations

$$c_o = \sqrt{\frac{\gamma}{\gamma - 1}} \sqrt{2P_1(t_o)\ \rho_1(t_o)} \sum_1^n A\ K_n \left(\frac{\bar{\bar{P}}_{ex_n}}{P_1(t_o)}\right)^{1/\gamma} \qquad (25)$$

and

$$c_1 = \sum_{1}^{n} \left(\frac{\bar{P}_{ex_n}}{P_1(t_o)}\right)^{\frac{\gamma-1}{\gamma}}, \tag{26}$$

we obtain for the mass flow rate of equation (22) finally

$$\sum_{n} \dot{m}_o = c_o \left(\frac{\bar{P}_1}{P_1(t_o)}\right)^{\frac{\gamma-1}{2\gamma}} \left[1 - \left(\frac{P_1(t_o)}{\bar{P}_1}\right)^{\frac{\gamma-1}{\gamma}} c_1\right]^{1/2}, \tag{27}$$

and with equation (21), equation (27) becomes

$$\left[\frac{\sum_{n} \dot{m}_o}{c_o}\right]^2 = \left[1 - X\right]^{\frac{\gamma-1}{\gamma}} - c_1. \tag{28}$$

The first term on the right-hand side of equation (28) can be developed into a series

$$\left[1 - X\right]^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma - 1}{\gamma} X - \frac{\gamma - 1}{2\gamma^2} X^2 - \frac{(\gamma - 1)(\gamma + 1)}{6\gamma^3} X^3 + O(X^4). \tag{29}$$

Inserting (29) into (28) yields

$$\left[\frac{\sum_{n} \dot{m}_o}{c_o}\right]^2 = 1 - c_1 - \frac{\gamma - 1}{\gamma} X - \frac{\gamma - 1}{2\gamma^2} X^2 - \frac{(\gamma - 1)(\gamma + 1)}{6\gamma^3} X^3 + O(X^4). \tag{30}$$

Similarly, we proceed with equation (20), where the outflowing mass $\dot{m}_o$ is brought to the left side. With the stipulation already mentioned above, i.e.,

$$\bar{P} = \frac{P(t_1) + P(t_o)}{2} ,$$

we obtain

$$\sum_n \dot{m}_o = \sum_n \dot{m}_i + \frac{\rho_1(t_o)V}{\Delta t}\left[1 - \left(2\,\frac{\bar{P}_1}{P_1(t_o)} - 1\right)^{1/\gamma}\right]. \tag{31}$$

With the substitution of

$$c_2 = \frac{1}{1 + \dfrac{\Delta t \sum \dot{m}_i}{\rho_1(t_o)V}} \tag{32}$$

and equation (21) inserted into equation (31) yields

$$\left[\frac{\Delta t\ c_2 \sum\limits_n \dot{m}_o}{P_1(t_o)\ V}\right]^2 = 1 - 2c_2\left[2(1 - X) - 1\right]^{1/\gamma} + c_2^2\left[2(1 - X) - 1\right]^{2/\gamma} . \tag{33}$$

The inflowing mass

$$\sum \dot{m}_i$$

in equations (31) and (32) is assumed to be known, since it represents the mass-flow of the upstream compartment for which the procedure described here has already been conducted. For $n(n > 1)$ compartment, however, a second iteration is necessary, since the outflowing mass, $\dot{m}_o$, influences the inflowing mass, $\dot{m}_i$, unless the upstream orifice was choked. Development into a series leads finally to

$$\left[\frac{\Delta t \; c_2 \sum\limits_{n} \dot{m}_o}{P_1(t_o) \; V}\right]^2 = (1 - c_2)^2 + \frac{4c_2X}{\gamma}(1 - c_2) + \frac{4c_2X^2}{\gamma^2}\Big[(\gamma - 1) + c_2(2 - \gamma)\Big]$$

$$+ \frac{(\gamma - 1)\; 8c_2X^3}{3\gamma^3}\Big[(2\gamma - 1) + 4c_2(2 - \gamma)\Big] + O(X^4). \quad (34)$$

Equations (34) and (30) have to be set equal in $\dot{m}_o$ to obtain X. Before we go to this step, however, some useful abbreviations are introduced. We set:

$$\alpha_0 = (1 - c_2)^2; \qquad \beta_0 = (1 - c_1)$$

$$\alpha_1 = 4c_2(1 - c_2)/\gamma; \qquad \beta_1 = (\gamma - 1)/\gamma$$

$$\alpha_2 = 4c_2[(\gamma - 1) + c_2(2 - \gamma)]/\gamma^2; \qquad \beta_2 = (\gamma - 1)/(2\gamma^2)$$

$$\alpha_3 = (\gamma - 1)\; 8c_2[(2\gamma - 1) + 4c_2(2 - \gamma)]/3\gamma^3; \qquad \beta_3 = (\gamma - 1)(\gamma + 1)/(6\gamma^3)$$

$$\alpha_4 = [\Delta t c_2/(\rho_1(t_o)V)]^2 \qquad \beta_4 = (1/c_o)^2$$

(35)

and

$$\beta'_o = \beta_o/\beta_4; \qquad \alpha'_o = \alpha_o/\alpha_4 \qquad A'_1 = \frac{\beta'_1 + \alpha'_1}{\beta'_2 + \alpha'_2}$$

$$\beta'_1 = \beta_1/\beta_4; \qquad \alpha'_1 = \alpha_1/\alpha_4 \qquad A'_o = \frac{\beta'_o - \alpha'_o}{\beta'_2 + \alpha'_2}$$

$$\beta'_2 = \beta_2/\beta_4; \qquad \alpha'_2 = \alpha_2/\alpha_4 \qquad A'_3 = \frac{\alpha'_3 + \beta'_3}{\beta'_2 + \alpha'_2}$$

$$\beta'_3 = \beta_3/\beta_4; \qquad \alpha'_3 = \alpha_3/\alpha_4$$

and the solution for X yields

$$X = -\frac{A_1'}{2} + \sqrt{A_o' - A_3' X^3 + (A_1'/2)^2}\,, \tag{36}$$

where A_3' can be left out and X can be obtained without iteration or can be used as a correction to X, if retaining of only the quadratic terms is not enough to obtain an adequate accuracy. However, only the positive sign of the square root is used for the mass flowing out of the compartment. The mean pressure of the compartment is finally

$$\bar{P}_1 = P_1(t_o)[1 - X]. \tag{37}$$

For choked flow, equation (17) is used, which becomes now, for n orifices of the compartment,

$$\left[\sum_n \dot{m}_o\right]^2 = \left(\frac{\bar{P}_1}{P_1(t_o)}\right)^{\frac{\gamma+1}{\gamma}} \sum_n (A_n K_n)^2 \left(\frac{2}{\gamma + 1}\right)^{\frac{2}{\gamma-1}} \frac{\gamma}{\gamma + 1}\left[2P_1(t_o)\,\rho_1(t_o)\right]. \tag{38}$$

With the abbreviation

$$c_o^* = \left(\frac{2}{\gamma + 1}\right)^{\frac{2}{\gamma-1}} \frac{\gamma}{\gamma + 1}\left[2P_1(t_o)\ \rho_1(t_o)\right] \sum_n (A_n K_n)^2, \tag{39}$$

equation (38) becomes

$$\left[\sum_n \dot{m}_o\right]^2 = c_o^*\left[1 - \frac{\gamma + 1}{\gamma} X + \frac{\gamma - 1}{2\gamma^2} X^2 + \frac{(\gamma + 1)(\gamma - 1)}{6\gamma^3} X^3 + O(X^4)\right]. \tag{40}$$

Substituting

$$\beta_o = 1; \qquad \beta_o' = \beta_o/\beta_4$$

$$\beta_1 = (\gamma + 1)/\gamma; \qquad \beta_1' = \beta_1/\beta_4$$

$$\beta_2 = -(\gamma + 1)/(2\gamma^2); \qquad \beta_2' = \beta_2/\beta_4$$

$$\beta_3 = -(\gamma + 1)(\gamma - 1)/(6\gamma^3); \qquad \beta_3' = \beta_3/\beta_4$$

$$\beta_4 = 1/c_o^*;$$

we obtain the same expression as equation (36).

B. The Adiabatic Flow in Constant Area Ducts with Friction

The assumption is again that the gas is perfect. The rate of change of the gas properties depends now on the amount of friction, so that the momentum equation must be introduced.

The perfect gas relation was given by equation (13); taking logarithmic differentials, we obtain

$$\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T} \, . \tag{41}$$

The definition of the Mach number, which was not previously introduced is, for a perfect gas,

$$M^2 = v^2/\gamma RT \tag{42}$$

or

$$\frac{dM^2}{M^2} = \frac{dv^2}{v^2} - \frac{dT}{T} \, . \tag{43}$$

The energy equation, given by (10) with $dQ = 0$ becomes by dividing through by c_pT and using the definition for the Mach number

$$\frac{dT}{T} + \frac{\gamma - 1}{2} M^2 \frac{dv^2}{v^2} = 0. \tag{44}$$

The continuity equation (12) becomes in terms of logarithmic differentials with the mass flow as a constant

$$\frac{d\rho}{\rho} + \frac{1}{2} \frac{dv^2}{v^2} = 0. \tag{45}$$

The momentum equation is written as

$$-AdP - \tau_x \, dA_x = \dot{m}dv, \tag{46}$$

where τ_x is the shearing stress exerted on the stream by the walls, and dA_x is the wetted wall area over which τ_x acts. Introducing a friction coefficient $f(x)$, we obtain

$$f = \frac{2\tau_x}{\rho v^2} \tag{47}$$

and a hydraulic diameter

$$D_H = \frac{4A}{dA_x/dX} = \frac{R_H}{2}. \tag{48}$$

For a circular pipe the hydraulic diameter is thus equal to the diameter of the pipe. Inserting these expressions into equation (46), we obtain the momentum equation which, divided through by P, can be written, with $\rho v^2 = \gamma P M^2$, as

$$\frac{dP}{P} + \frac{\gamma M^2}{2} 4f \frac{dX}{D_H} + \frac{\gamma M^2}{2} \frac{dv^2}{v^2} = 0.$$

By eliminating dP/P and dv^2/v^2, we obtain finally, for a pipe of length L between sections 0 and L

$$\int_0^L 4f(X)\,\frac{dX}{D_H} = \int_{M_0^2}^{M_1^2} \frac{1 - M^2}{\gamma (M^4)\left(1 + \frac{\gamma - 1}{2} M^2\right)}\, dM^2. \tag{49}$$

With a mean friction factor $\bar{f}$ between stations 0 and L as

$$\bar{f} = \frac{1}{L} \int_0^L f dX. \tag{50}$$

Equation (49) when integrated between 0 and L or M_0^2 and M_1^2 becomes

$$\gamma 4\bar{f}\,\frac{L}{D_H} = \left[\frac{1}{M_0^2} - \frac{1}{M_1^2} + \frac{\gamma + 1}{2}\ \ln \frac{M_0^2\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{M_1^2\left(1 + \frac{\gamma - 1}{2} M_0^2\right)}\right]. \tag{51}$$

Known data are the friction coefficient $\bar{f}$ for the pipe of length L and the hydraulic diameter D_H. The inlet Mach number M_0^2 can be computed with the aid of the mass flow from the upper compartment to the pipe and the pressure at the pipe inlet, depending on the type of approach: isentropic for a converging pipe inlet section or adiabatic for a sudden constriction. The pressure, not known a priori, can only be obtained by an iteration scheme, assuming first a Mach number, M_0. Then with equation (51) the Mach number increase for subsonic flow can be calculated between 0 and 1. With the relation

$$P_1 = P_0\,\frac{M_0}{M_1}\sqrt{\frac{\left(1 + \frac{\gamma - 1}{2} M_0^2\right)}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}}\,,$$

the respective pressure drop due to friction can be calculated. If the Mach number at station 1 is subsonic, then the exit pressure P_1 must be equal to P_{ex}, the atmospheric pressure outside the pipe. If this is not the case, we must repeat the calculation with another P_o, which in turn will alter the mass flow from the upstream compartment.

If the exit of the pipe is choked, $M_1 = 1$, then the Mach number M_1 is fixed and one has to calculate on an iterative basis the inlet Mach number, M_o, and the corresponding pressure, P_o. The pressure and density at station 0 are then given by the relations

$$\frac{P_o}{P^*} = \frac{1}{M_o}\sqrt{\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M_o^2\right)}}, \tag{52}$$

and

$$\frac{v_o}{v^*} = \frac{\rho^*}{\rho_o} = M_o\sqrt{\frac{\gamma + 1}{2\left(1 + \frac{\gamma - 1}{2} M_o^2\right)}} \tag{52a}$$

where the asterisks designate critical values for which the Mach number is $M = 1$.

Equation (51) cannot be solved explicitly. For subsonic flow, the following iteration scheme has proven to be stable.

$$M_1 = \frac{M_o}{\left[1 + M_o^2 \frac{\gamma + 1}{2} \ln \frac{M_o^2\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{M_1^2\left(1 + \frac{\gamma - 1}{2} M_o^2\right)} - M_o^2\gamma\left(4\bar{f}\frac{L}{D} + \zeta\right)\right]^{1/2}}, \tag{53}$$

where ζ is a loss coefficient of obstacles in the pipe such as bends, constrictions, etc. It can be a function of the Mach number or the geometry of the pipe. In the equation M_1 appears in the logarithmic term of the denominator. The length of the pipe for choked flow is given by

$$L = \frac{D_H}{4\gamma\bar{f}}\left[\frac{1}{M_o^2} - 1 + \frac{\gamma + 1}{2}\ln\frac{M_o^2\,(1+\gamma)}{2\left(1 + \frac{\gamma - 1}{2}M_o^2\right)} - \gamma\zeta\right]. \tag{54}$$

C. Flow in Ducts with Heating or Cooling and Friction

The determination of the flow parameters again requires the application of the three conservation equations and the perfect gas equation. We retain now dQ in equation (10):

$$dQ = c_p dT + d(v^2/2) = c_p dT_t, \tag{55}$$

and with the definition of the Mach number of a perfect gas equation (42), we can find the following relations [8]:

For the Mach number increase along the pipe,

$$\frac{dM^2}{M^2} = \frac{(1 + \gamma M^2)\left(1 + \frac{\gamma - 1}{2}M^2\right)}{1 - M^2}\frac{dT_t}{T_t(x)} + \frac{\gamma M^2\left(1 + \frac{\gamma - 1}{2}M^2\right)}{1 - M^2}\,4\bar{f}\,\frac{d(x/L)}{(D_H/L)}. \tag{56}$$

For the velocity increase,

$$\frac{dv}{v} = \frac{1 + \frac{\gamma - 1}{2}M^2}{1 - M^2}\frac{dT_t}{T_t(x)} + \frac{\gamma M^2}{2(1 - M^2)}\,4\bar{f}\,\frac{d(x/L)}{(D_H/L)}. \tag{57}$$

For the speed of sound,

$$\frac{dc}{c} = \frac{(1 - \gamma M^2)\left(1 + \frac{\gamma - 1}{2}M^2\right)}{2(1 - M^2)}\frac{dT_t}{T_t(x)} - \frac{\gamma(\gamma - 1)M^2}{4(1 - M^2)}\,4\bar{f}\,\frac{d(x/L)}{(D_H/L)}. \tag{58}$$

For the temperature,

$$\frac{dT}{T} = \frac{(1 - \gamma M^2)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2} \frac{dT_t}{T_t(x)} - \frac{\gamma(\gamma - 1) M^4}{2(1 - M^2)} 4\bar{f} \frac{d(x/L)}{(D_H/L)}. \tag{59}$$

For the density decrease along the pipe,

$$\frac{d\rho}{\rho} = - \frac{1 + \frac{\gamma - 1}{2} M^2}{1 - M^2} \frac{dT_t}{T_t(x)} - \frac{\gamma M^2}{2(1 - M^2)} 4\bar{f} \frac{d(x/L)}{(D_H/L)}. \tag{60}$$

For the pressure decrease,

$$\frac{dP}{P} = - \frac{\gamma(M^2)\left(1 + \frac{\gamma - 1}{2} M^2\right)}{1 - M^2} \frac{dT_t}{T_t(x)} - \frac{\gamma M^2 \left(1 + (\gamma - 1) M^2\right)}{2(1 - M^2)} 4\bar{f} \frac{d(x/L)}{(D_H/L)}. \tag{61}$$

The total pressure loss is given by

$$\frac{dP_t}{P_t} = - \frac{\gamma M^2}{2} \frac{dT_t}{T_t(x)} - \frac{\gamma M^2}{2} 4\bar{f} \frac{d(x/L)}{(D_H/L)}. \tag{62}$$

Equations (56) to (62) have to be integrated numerically. Q is the heat transferred to the flow within the tube from external sources, per unit mass of stream.

Equation (56) shows that heat addition, expressed as total temperature increase, causes the Mach number to increase above that caused by the friction force. The choking length of the tube is therefore shorter. We notice that equation (56) is a singular ordinary differential equation of the first order. In the domain $0 \leq M^2 < 1$, the right-hand side of equation (56) is continuous. A singular point is encountered at $M^2 = 1$.

We solve the equation by applying the method of successive approximation, which is also known as Picard's method. If equation (56) is written in the general form as

$$\frac{dM^2}{dx} = f(M^2, x),$$

then the solution has the form of

$$M^2(x) = M^2(x_o) + \int_{x_o}^{x} f(M^2, \xi)\, d\xi.$$

This relation is, in reality, an integral equation, involving the dependent variable under the integral sign. It can be solved on an iterative basis as follows:

$$M_1^2(x) = M^2(x_o) + \int_{x_o}^{x} f(\xi, M_o^2)\, d\xi$$

$$M_2^2(x) = M^2(x_o) + \int_{x_o}^{x} f(\xi, M_1^2)\, d\xi$$

$$M_n^2(x) = M^2(x_o) + \int_{x_o}^{x} f(\xi, M_{n-1}^2)\, d\xi.$$

As n increases indefinitely, the sequence of functions $M_n^2(x)$ tends to a limit which is a continuous function of x, and the limit-function satisfies the differential equation. On the other hand, the Lipschitz condition has to be satisfied. If (x, M_1^2) are two points within the domain of the same abscissa, then

$$|f(x,M_2^2) - f(x,M_1^2)| < K|M_2^2 - M_1^2|,$$

where K is a constant. In the neighborhood of M = 1, the Lipschitz condition no longer holds. Rearranging equation (56) yields

$$\frac{dx}{dM^2} = \frac{1 - M^2}{M^2\left(1 + \frac{\gamma - 1}{2} M^2\right)\left[(1 + \gamma M^2) \frac{dT_t}{d(x/L)} \frac{1}{T_t(x)} + \gamma M^2 4\bar{f} \frac{L}{D}\right]},$$

or in general form

$$\frac{dx}{dM^2} = g(x,M^2).$$

The function $g(M^2,x)$ is now singular at $M^2 = 0$ and $dx/dM^2 = 0$ for $M^2 = 1$. The method of successive approximations can again be applied. The solution now has the form

$$x(M^2) = x(M_o^2) + \int_{M_o^2}^{M^2} g(x,M^2)\, dM^2.$$

Instead of the Lipschitz condition, a less stringent condition can be applied. For instance, a step sensitivity condition such as

$$\left|\frac{dM^2}{dx}\right| \leq \epsilon_1; \qquad |M_1^2 - M_o^2| \leq \epsilon_1|x_1 - x_o|$$

for the first case and

$$\left|\frac{dx}{dM^2}\right| \leq \epsilon_2; \qquad |x_1 - x_o| \leq \epsilon_2|M_1^2 - M_o^2|.$$

If ϵ_1 and ϵ_2 are of the order $\epsilon_1;\epsilon_2 \leq .002$, the analytic solution of equation (56) with $dT_t = 0$ was almost exactly reproduced. The two solutions are joined where the step sensitivity condition was satisfied for both regions.

Once the Mach number distribution along the length of the pipe is obtained, the other flow properties can be calculated by a simple relation of the properties at two stations (1 and 2):

$$\frac{P_2}{P_1} = \frac{M_1}{M_2}\sqrt{\frac{T_{t2}}{T_{t1}}\frac{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)}} \; ,$$

$$\frac{V_2}{V_1} = \frac{M_2}{M_1}\sqrt{T_2/T_1} \; ,$$

$$\frac{P_2}{P_1} = \frac{P_2 T_1}{P_1 T_2} \; ,$$

$$\frac{T_2}{T_1} = \frac{T_{t2}}{T_{t1}}\frac{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)}{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)} \; ,$$

$$\frac{P_{t2}}{P_{t1}} = \frac{P_2}{P_1}\left[\frac{1 + \frac{\gamma - 1}{2} M_2^2}{1 + \frac{\gamma - 1}{2} M_1^2}\right]^{\frac{\gamma}{\gamma - 1}} \; .$$

The Mach number distribution in general depends on the nature of the temperature distribution and the heat flux along the pipe. For some cases, one can combine the friction coefficient and heat transfer coefficient via the Reynolds analogy, and the integration of equation (56) might be possible in closed form [8].

D. Sudden Enlargement in a Pipe

The only loss coefficient which can be calculated by simple analytical methods is the case where there is a sudden enlargement in a pipe. The problem was first treated by Borda and Carnot and is therefore known as the "Borda-Carnot loss."

The flow is again adiabatic, and it is assumed that the pressure across the face of the enlargement is equal to the pressure in the smaller pipe, just before the enlargement.* According to Figure 1b, the three basic equations can be written as follows:

Momentum

$$\dot{m}(v_2 - v_1) = A_2(P_1 - P_2). \tag{63}$$

Continuity

$$P_1 v_1 A_1 = \rho_2 v_2 A_2. \tag{64}$$

Energy

$$\frac{v_1^2}{2} + \frac{\gamma}{\gamma - 1}\frac{P_1}{\rho_1} = \frac{v_2^2}{2} + \frac{\gamma}{\gamma - 1}\frac{P_2}{\rho_2} = \frac{\gamma + 1}{2(\gamma - 1)} c^{*2}, \tag{65}$$

where c^* is the velocity of sound where the Mach number is unity. For adiabatic flow, the stagnation temperature is constant, and we can write for $v = c^*$, the right-hand term of equation (65) as

$$\frac{P_1}{\rho_1} = \frac{\gamma - 1}{\gamma}\left[c^{*2}\frac{\gamma + 1}{2(\gamma - 1)} - \frac{v_1^2}{2}\right] = \frac{\gamma + 1}{2\gamma} c^{*2} - \frac{\gamma - 1}{2\gamma} v_1^2 \tag{66}$$

and

$$\frac{P_2}{\rho_2} = \frac{\gamma + 1}{2\gamma} c^{*2} - \frac{\gamma - 1}{2\gamma} v_2^2. \tag{67}$$

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*This assumption restricts the subsequent equations and $M_1 \leq 1$.

From equation (63) and (64), we obtain

$$v_1 - v_2 = \frac{(P_2 - P_1)\, A_2}{\rho_2 v_2 A_2} = \frac{\rho_2}{\rho_2 v_2} - \frac{P_1}{\rho_1 v_1 \emptyset}. \tag{68}$$

Hence, with equation (52) and (67) introduced into (68)

$$\frac{v_1 - v_2}{c^*} = \frac{c^*}{v_2}\left\{\frac{\gamma + 1}{2\gamma} - \frac{\gamma - 1}{2\gamma}\,\frac{v_1^2}{c^{*2}}\right\} - \frac{c^*}{\emptyset v_1}\left\{\frac{\gamma + 1}{2\gamma} - \frac{\gamma - 1}{2\gamma}\,\frac{v_1^2}{c^{*2}}\right\}, \tag{69}$$

where $\emptyset$ is equal to the ratio A_1/A_2. In terms of the critical Mach number M^*, we obtain

$$M_1^* - M_2^* = \frac{1}{M_2^*}\left\{\frac{\gamma + 1}{2\gamma} - \frac{\gamma - 1}{2\gamma}\,M_2^{*2}\right\} - \frac{1}{\emptyset M_1^*}\left\{\frac{\gamma + 1}{2\gamma} - \frac{\gamma - 1}{2\gamma}\,M_1^{*2}\right\} \tag{70}$$

and

$$(M_2^*)^2 + \left\{\frac{1}{\emptyset}\,\frac{\gamma - 1}{\gamma + 1}\,M_1^* - \frac{2\gamma}{\gamma + 1}\,M_1^* - \frac{1}{\emptyset M_1^*}\right\} M_2^* + 1 = 0. \tag{71}$$

This equation is solved for M_2^*, the Mach number in the larger pipe, and the positive sign of the square root is taken. The density and pressure ratios can then be calculated as

$$\text{density} - \quad \frac{\rho_2}{\rho_1} = \frac{M_1^*\,\emptyset}{M_2^*} \tag{72}$$

and, with the gas law (13),

$$\text{pressure} - \quad \frac{P_2}{P_1} = \frac{M_1^*\left(1 - \frac{\gamma - 1}{\gamma + 1}\,M_2^{*2}\right)}{M_2^*\left(1 - \frac{\gamma - 1}{\gamma + 1}\,M_1^{*2}\right)}. \tag{73}$$

The relationship between Mach number M and M^*, is

$$M^2 = \frac{2M^{*2}}{(\gamma + 1) - (\gamma - 1) M^{*2}}. \tag{74}$$

The density and pressure can be written as follows:

$$\text{density - } \quad \frac{\rho_2}{\rho_1} = \emptyset \frac{M_1}{M_2} \sqrt{\frac{(\gamma - 1) M_2^2 + 2}{(\gamma - 1) M_1^2 + 2}} . \tag{75}$$

$$\text{pressure - } \quad \frac{P_2}{P_1} = \emptyset^2 \frac{M_1^2}{M_2^2} \frac{\rho_1}{\rho_2} . \tag{76}$$

E. A Sudden Contraction in a Pipe

Here, the main features are an acceleration zone leading to the development of a vena contracta, followed by a deceleration zone similar to that analyzed by Borda and Carnot.

The total pressure loss can be calculated approximately by the following idealization. The smaller tube with the area A_2 and a given mass flow $\dot{m}$ is placed into a stream of velocity $V_1 < V_2$. Then the stagnation point of the free streamline is located on the outside of the tube. The gas contained inside the free stream tube of area A_1 flows with an energy loss through the tube, since the flow separates at the lip of the pipe and contracts at the entrance (see Figure 10, lower part of the schematic). The surface of the free stream tube can now be solidified, thus forming the upstream tube with the area A_1; ($A_1 > A_2$).

The momentum equation in the x-direction yields

$$\dot{m} V_2 + (P - P_1)_2 A_2 = \dot{m} V_1 + (P - P_1)_1 A_1 + S + \int_S (P - P_1) \, dS_x . \tag{77}$$

If we assume that the wall of the narrow pipe is thin $\delta/d_2 \approx 0$, then the suction force S can be taken as zero. The pressure difference $(P - P_1)$ integrated over the free stream tube surface is zero by definition of the free streamline. However, in practical cases, where the idealization is rather far off, a pressure difference $(P - P_1)$, dependent on length b, builds up in the corner of the two pipes BCDE. With decreasing length b, the difference in the pressures P and P_1 becomes more pronounced and the integral must be taken into account. One approximation could be obtained for the case of $b = 0$ and $A_1 \gg A_2$.

Abbreviating equation (77), we get

$$T = S + \int_S (P - P_1)\, dS_x .$$

The mass flow through the tube is constant and

$$\dot{m} = \rho_2 V_2 A_2 = \gamma P_2 \frac{M_2}{c_t} (c_t/c)_2 A_2 . \tag{78}$$

With the continuity equation,

$$P_2 M_2 (c_t/c)_2 A_2 = P_1 M_1 (c_t/c)_1 A_1 , \tag{79}$$

and the relation,

$$\frac{P_{t2}}{P_{t1}} = \frac{P_1/P_{t1}}{P_2/P_{t2}} \cdot \frac{P_2}{P_1} , \tag{80}$$

one can obtain, after rearranging equation (77) and solving for

$$\frac{P_2}{P_1} = \frac{1}{1 + M_2^2 - M_2 M_1 (c_t/c_2)/(c_t/c_1) - T/(P_2 A_2)} ,$$

the expression for the total pressure loss across the contraction

$$\frac{P_t}{P_t} = \frac{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{\frac{\gamma}{\gamma-1}}}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}} \cdot \frac{1}{1 + \gamma M_2^2 - \gamma M_1 M_2 \frac{\left(1 + \frac{\gamma - 1}{2} M_2^2\right)^{1/2}}{\left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{1/2}} - \frac{T}{P_2 A_2}} \cdot \tag{81}$$

The term T/P_2A_2 is definitely a function of M_2^2 and M_1^2. For the case of $M_1 = 0$, a very simple expression is obtained for calculating the Mach number M_2. We set

$$\alpha = \frac{\dot{m}}{A_2^2\, \gamma\, P_{t1}\, \rho_{t1}}\ ; \qquad \beta = \gamma \left[1 - \frac{T}{P_2\, A_2\, \gamma\, M_2^2}\right]. \tag{82}$$

Then the Mach number in the pipe entrance becomes

$$M_2^2 = \frac{(2\beta\alpha - 1)}{(\gamma - 1 - 2\beta^2\alpha)} \pm \sqrt{\frac{2\alpha}{(\gamma - 1 - 2\beta^2\alpha)} + \frac{(2\beta\alpha - 1)^2}{(\gamma - 1 - 2\beta^2\alpha)^2}}\ , \tag{83}$$

where the M_2^2 is chosen, which is positive and smaller than unity. In engineering literature where energy loss coefficients are considered, the definition is

$$\zeta = \frac{2\Delta E}{V_2^2} = \frac{P_1 - P_2}{q_2} + \frac{V_1^2 - V_2^2}{V_2^2} = \frac{P_{t1} - P_{t2}}{q_2}$$

and in terms of the total pressure ratio between the stations 1 and 2 in Figure 1C,

$$\zeta = \frac{P_{t1} - P_{t2}}{\frac{\gamma}{2}\, P_2\, M_2^2} = \frac{2}{\gamma M_2^2}\, (P_{t1}/P_1)\, \frac{P_1}{P_2}\, \left(1 - \frac{P_{t2}}{P_{t1}}\right)$$

where

$$\frac{P_{t1}}{P_1} = \left(1 + \frac{\gamma - 1}{2} M_1^2\right)^{\frac{\gamma}{\gamma-1}}; \qquad q_2 = \frac{\gamma}{2}\, P_2\, M_2.$$

When we develop P_{t1}/P_1 into a series for small M_2 and let A_1 increase indefinitely, so that $M_1 = 0$, we obtain a simple expression for the energy loss coefficient

$$\zeta = \left(1 - \frac{1}{4} M_2^2 - \frac{2 - \gamma}{24} M_2^4 - \frac{2}{\gamma M_2^2 P_2 A_2} \frac{T}{P_2 A_2} - 0(M_2^6)\right) .$$

Our next task is to obtain an expression for the integral term T. For a sharp-lipped inlet of the pipe, the suction force S vanishes, and thus

$$T = \int_S (P - P_1) \, dS_x .$$

In general, the pressure difference along the distance C - D in Figure 1C is a function of M_1 and M_2. In our case, however, $M_1 = 0$, and therefore the term

$$\frac{2T}{\gamma M_2^2 P_2 A_2} = 2 \int_\infty^1 c_p(r/R_2)(r/R_2) \, d(r/R_2)$$

is only a function of the Mach number M_2 in the pipe. As a next step, we want to obtain an approximate expression for the pressure coefficient along the wall C - D. As a first approximation, which is valid for small Mach numbers M_2, we treat the flow as incompressible, $\rho_1 = \rho_2 = \rho$. The length b is zero. The Bernoulli equation along the streamline C - D yields

$$\frac{V_1^2}{2} + \frac{P_1}{\rho_1} = \frac{V_2^2}{2} + \frac{P_2}{\rho_2} \frac{V^2}{2} + \frac{P}{\rho} ,$$

and the pressure coefficient becomes

$$c_p = \frac{V_1^2 - V^2}{V_2^2} .$$

For $A_1 \rightarrow \infty$, the velocity V_1 vanishes, and the pressure coefficient reduces to

$$c_p = - V^2/V_2^2 .$$

The mass flow is constant between stations 1 and C, where $A_c = K_o A_2$ and K_o is the contraction coefficient

$$\dot{m} = \rho \, V_C \, K_o \, A_2 = \rho \, V_2 \, A_2$$

and therefore

$$V_C = \frac{V_2}{K_o} .$$

The limit of the pressure coefficient between point C, which has shifted with $A_1 \to \infty$, and D is at point C $V = V_1 = 0$, and the pressure coefficient vanishes. At point D, $V \approx V_C = V_2/K_o$ and the pressure coefficient becomes $c_p = -1/K_o$. Between the points C and D, the velocity behaves, at large distances from D, as a point source in the presence of a wall, and the velocity is proportional to $1/(r/R_2)^2$. In the vicinity of D, $r = R_2$, the velocity vanishes faster than $1/(r/R_2)^2$, since the derivative of V at R_2 is infinite; V, however, is finite and equal to V_C. The velocity derivative dV/dS, as a matter of fact, has theoretically a logarithmic discontinuity. If we still apply an exponential law to

$$v(r/R_2) = V_C \frac{1}{(r/R_2)^n} ,$$

the exponent n should grow indefinitely for $r \to R_2$.

If we accept the value of $\zeta \approx .5$ for a sharp-edged orifice and for the length b = 0, as all engineering handbooks propose, an average value for

$$n = \frac{2 + K_o^2}{K_o^2}$$

will be obtained. For a vena contracta in a confined space, the pressure in the separation bubble is lower than in the atmosphere, tending to increase the contraction coefficient, K_o. According to figure 2, a value of $K_o \approx .8$ is more appropriate than $K_o \approx .62$, which is the value for a jet issuing from a flat-plate orifice into the atmosphere. With the value of $K_o \approx .8$, the exponent becomes n = 4.125, which seems to be reasonable. We choose n = 4.0, and the expression for the integral term becomes

$$\frac{T}{P^2 A_2} = \frac{\gamma}{2} M_2^2 \frac{1}{3K_o^2} .$$

III. THE ANALYTIC REPRESENTATION OF THE DISCHARGE COEFFICIENT AND OTHER FLOW AND ATMOSPHERIC PROPERTIES

An analytical investigation of the rather complex flow problems requires that the various coefficients of discharge, pipe friction, and the atmospheric properties, such as pressure and density, as a function of altitude, be represented in closed form, or given as tables. For efficiency, a closed analytical form of representation is favored.

A. Discharge Coefficient

The discharge coefficient deviated remarkably from unity primarily because of the contraction of the streamlines. The streamlines curve or converge as they approach the orifice opening, bend around the edge of the hole and continue to curve and converge beyond the orifice. At some distance from the plane of the orifice, the het has a minimum section at which the streamlines are parallel. Because of the friction, the actual jet velocity in the orifice is less than the ideal jet velocity based on isentropic flow. A combination of these two factors is the discharge coefficient. The coefficient increases substantially as the result of compressibility as shown in Figure 2. If orifices are built into a pipe, the discharge coefficient depends also on the diameter ratio of the orifice and upstream pipe [5]. With orifice-to-pipe diameter approaching one, the constriction of the flow disappears, and the discharge coefficient will also attain the value of one (Figure 3).

Another important parameter is the Reynolds number formed by the orifice diameter and approach velocity. For small Reynolds numbers below 200, the discharge coefficient K decreases markedly as can be seen in Figures 4 and 5. Leak flows fall definitively into this region.

Some of the curves in Figure 2 represent the free jet; i.e., the jet discharging into still air. (These curves are obtained from References 1 and 3.) A dependence on the aspect ratio for square openings is noticeable. For comparison, the discharge coefficient of a circular sharp edge orifice is also drawn. For the pressure ratio, P_{ex}/P, approaching one, the coefficient has approximately the value of the theoretical calculations of the vena contracta of about $K_o \approx .6$ - .61 by Treffts and .58 by Garabedian - [5]. The coefficients for two- and three-dimensional flow are probably the same. However, since the combined effect of friction and contraction is plotted, the difference can be attributed mostly to the first effect. The compressible jets have a higher curvature at the orifice [11], thus increasing the minimum cross section of the jet far downstream of the orifice.

For the subprogram, the discharge coefficient K_o was approximated as

$$K_o = 1. - .155 \exp\left(-3.5(P_{ex}/P_i)\right) - .5186 \frac{\exp(P_{ex}/P_i) - \exp(-P_{ex}/P_i)}{\exp(P_{ex}/P_i) + \exp(-P_{ex}/P_i)}, \quad (84)$$

which gives a fair value over the range $0 \leq P_{ex}/P_i \leq 1$.

Figure 3 shows the change of the discharge coefficient obtained from Reference 5. At $d/D = 1$, the discharge coefficient is $K_o = 1$, decreasing monotonically with d/D as predicted by theory to the value of $K_o = .6$, the discharge coefficient of an orifice cut into an infinite wall. The theoretical value is obtained at approximately $d/D = .23$; then gravity effects become probably more pronounced for liquid jets. A fair approximation is given by the expression

$$K = .6 + .4 \exp\left\{-(5.5 - \frac{d}{D})(1 - \frac{d}{D})\right\}. \quad (85)$$

In Figure 4 the discharge coefficientsfor plate orifices are drawn [10], showing the influence of the Reynolds number for various orifice-to-pipe diameter ratios. The Reynolds number is formed with the diameter and the approach velocity.

The discharge coefficients for close-clearance orifices are drawn in Figure 5 for various Reynolds numbers obtained from Reference 6. The Reynolds number is here defined as

$$R_e = \frac{\dot{m} R_H}{A\mu}, \quad (86)$$

where the hydraulic radius is given by

$$R_H = \frac{\text{area of flowing fluid}}{\text{wetted perimeter}}\ 2.$$

For a radial clearance, C, over flow length, L, approaching zero, the discharge coefficient for all Reynolds numbers decrease. For $C/L \geq 1$, the coefficient is only a function of the Reynolds number.

A general approximate expression for K/K_o as a function of C/L and R_e (Figure 5) is given by

$$\frac{K}{K_o} = \left(1 - e^{-4.788\sqrt{C/L}}\right)(a + n \ln R_e). \tag{87}$$

The constants differ for different Reynolds numbers and are given by:

$1 < R_e \leq 100$	$a = .02$	$n = .1867$
$100 < R_e \leq 500$	$a = .52$	$n = .0782$
$R_e > 500$	$a = 1.0$	$n = 0.$

(The hump in the curves at low C/L and high R_e numbers is not considered in these expressions.)

The coefficient K_o can be set equal to .62 without considering any effect of the outside flow.

When the jet issues into the outside stream, flowing normally to the jet axis, a further drop of the discharge coefficient is expected. In Figure 6 the discharge coefficient ratio, K/K_o, is drawn for an orifice [1] of aspect ratio 1 and for three different free stream Mach numbers over the mass flow rate ratio (m; $K_o/m_\infty K$). A definite influence of the free stream Mach number M_∞ can be observed. In Reference 1 the test was conducted with orifices of different aspect ratios and shapes cut into a wall placed tangentially to the outer flow field. At the position of the orifice, the boundary layer was relatively thin. With thicker boundary layer flow regions in the vicinity of the orifice, the discharge coefficient K should be somewhat improved. Further tests are necessary in this area.

For the venting problem, a mean representative curve was derived. Its expression is

$$\frac{K}{K_o} = \frac{X^n}{(1 - X)E + X^n} ; \quad X = \frac{m_i K_o}{m_\infty K} = \frac{\rho v_i K_o}{\rho_\infty v_\infty}, \tag{88}$$

where n = 4.5714 and E = .0101 representing the curve for M = 1 closely.

To calculate the proper discharge coefficient, it is necessary to determine the absolute viscosity of the discharging gas. In general, the viscosity, μ, is a function of the pressure and temperature. However, as the pressure and thus the density of the gas decreases, the absolute viscosity approaches the low density limit and the pressure dependence becomes less. For most gases, this low density limit is reached at a pressure of about one atmosphere. Since the pressure in the compartment is about one atmosphere at the beginning of venting and decreases further with time, we consider therefore only the temperature dependence.

The dependence of the viscosity on the temperature, obtained from Reference 4, is plotted in Figure 7 as a mineral plot of all gases. It gives the relationship between the reduced viscosity, $\mu_R = \mu/\mu_c$, the reduced temperature, $T_R = T/T_c$ with the reduced pressure $P_r = P/P_c$ as parameter; μ_c, T_c, and P_c are the critical viscosity, temperature, and pressure, respectively, of the particular gas under consideration.

Experimental values of μ_c are seldom available. However, μ_c may be estimated in the following way:

$$\mu_c = 7.7 \; M^{*1/2} \; P_c^{2/3} \; T_c^{-1/6} \; \times 10^{-7} \quad [\text{Kg/m sec}], \tag{89}$$

where P_c is the critical pressure in atm (Kg/cm^2) and T_c is the critical temperature in °K.

The dependence of μ_R on T_c is in general

$$\mu_R = (a \; T_R)^n, \tag{90}$$

where the constants are for the regions

$T_R \leq 1.5$	$a = .442$	$n = .9882$
$1.5 < T_R \leq 2.0$	$a = .462$	$n = .8204$
$2.0 < T_R \leq 3.0$	$a = .500$	$n = .6480$
$3.0 < T_R \leq 5.0$	$a = .564$	$n = .5185$
$5.0 < T_R \leq 10.0$	$a = .631$	$n = .5099.$

B. The Atmospheric Properties

For the calculation of the atmospheric properties of air, the formula and constants of Reference 9 were used. However, the rather complex expression for the acceleration due to gravity was further approximated by

$$g = g_o \frac{a^2}{[a + z]^2} + \ldots , \tag{91}$$

where a = 6,378,178 meters is the radius of the earth for the geographic latitude $\emptyset = 45°$ and z is the height above the earth's surface. The geopotential altitude becomes then

$$H = \int \frac{g}{g_o} az. \tag{92}$$

Integration yields

$$H = a \left[\frac{z}{a + z} \right]. \tag{93}$$

A comparison of the values obtained by this rough formula with the values of Reference 9 showed an error of 3 meters for an altitude of z = 70,000 meters. Other properties are obtained by the following formulas:

Pressure

$$P = P_b \left[\frac{T_{mb}}{T_{mb} + L'(H - H_o)}\right]^{\frac{g_o M_o}{R^* L'_m}}, \quad \text{for } L'_m \neq 0 \tag{94}$$

or

$$P = P_b \exp\left[-\frac{g_o H_o h}{R^* T_{mb}}\right], \quad \text{for } L'_m = 0. \tag{95}$$

Density

$$\rho = \frac{M_o P}{R^* T_m}. \tag{96}$$

Temperature

$$T = T_{mb} + L'_m(H - H_b). \tag{97}$$

Speed of Sound

$$c_s = \left(\gamma \frac{R^*}{M_o} T_m\right)^{1/2}. \tag{98}$$

In these equations the subscript b designates the respective values at the base of the sublayer. The constants for all layers are

g_o = 9.80665 m/sec^2

M_o = 28.9644 Kg/K mol

R^* = 8.31432 Joules(°K)$^{-1}$/mol.

The constants for the different sublayers are

$H = 0 \div 11$ Km

$L'_m = -6.5$; $T_{mb} = 288.15°K$; $H_b = 0m$; $P_b = 1.013250 \times 10^5 Newton/m^2$

$H = 11 \div 20$ Km

$L'_m = 0$; $T_{mb} = 216.65°K$; $H_b = 11000$ m; $P_b = 2.26320 \times 10^2$ mb

$H = 20 \div 32$ Km

$L'_m = 1$; $T_{mb} = 216.65°K$; $H_b = 20000$ m; $P_b = 5.47487 \times 10$ mb

$H = 32 \div 47$ Km

$L'_m = 2.8$; $T_{mb} = 228.65°K$; $H_b = 32000$ m; $P_b = 8.68014$ mb

$H = 47 \div 52$ Km

$L'_m = 0$; $T_{mb} = 270.65°K$; $H_b = 47000$ m; $P_b = 1.10905$ mb

$H = 52 \div 61$ Km

$L'_m = -2$; $T_{mb} = 270.65°K$; $H_b = 52000$ m; $P_b = 5.90005$ mb, etc.

C. The Friction Factor

In the equations for pipe flow, the friction factor, f, appears; accurate numerical values for this factor will be needed for the solution of engineering problems which require information on energy loss. Notice that f is an experimental coefficient usually determined by energy losses, the length and diameter of the pipe, and the velocity.

Nicuradse [10] conducted systematic tests, using measurable roughness produced by uniform sand grains of diameter e. Through the use of such artificial roughness, he was able to show that the friction factor depended upon Reynolds number and the relative roughness e/d (Figure 8). In this diagram the friction factor, f, is already the total value for a pipe of length, L. The Reynolds number therefore uses the total length, L. At high Reynolds number and high relative roughness, the friction factor becomes wholly a function of the relative roughness, e/d. Colebrooh and White [10] were able to correlate the results of many laboratory tests on commercial pipes over a wide range of Reynolds number and roughness by the following expression:

$$\frac{1}{\sqrt{4\bar{f}}} - 2\log_{10}\frac{d}{e} = 1.14 - 2\log_{10}\left[1 + \frac{9.28}{R_e(e/d)\sqrt{4\bar{f}}}\right]. \qquad (99)$$

However, care must be taken in defining relative roughness. For instance, if a pipe has a definite "regular" roughness, like a wavy wall and otherwise a smooth surface, then the curve would show a pronounced tendency to vary with the Reynolds number and a friction factor for a hydraulically smooth pipe has to be taken.

For a fully developed velocity profile, i.e., at distances from the pipe inlet greater than 50 diameters, no significant effect of Mach number was observed. The pipe Reynolds number is given as

$$R_e = v\rho D g/\mu.$$

For short pipes near the duct inlet, the one-dimensional analysis is misleading, since the velocity profile is not yet fully developed and changes in momentum flux of appreciable magnitude may occur. At the entrance, the flow is rather similar to the flat-plate flow, and the Reynolds number based on the flow length from the inlet is more important.

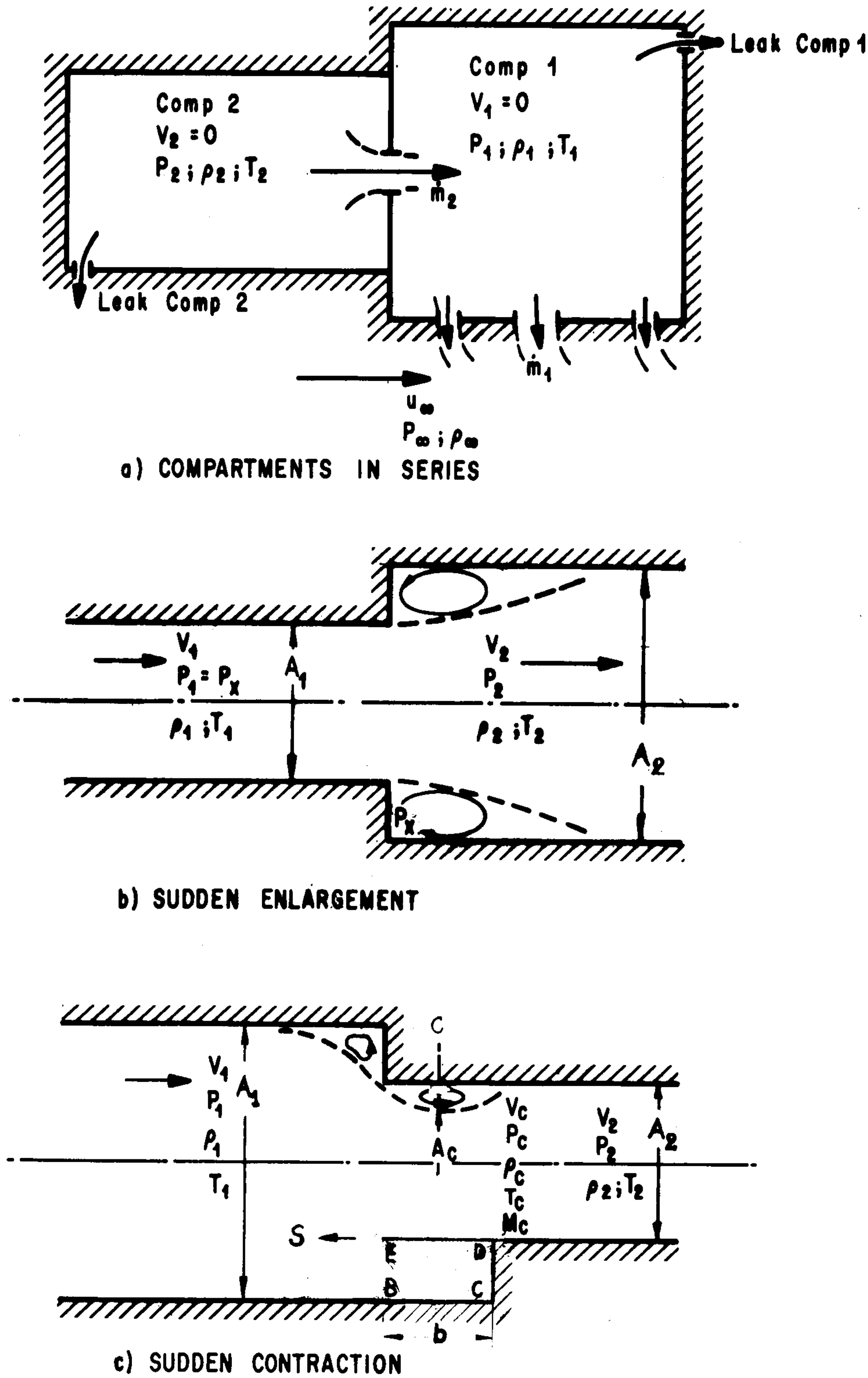


FIG. 1. SCHEMATIC OF POSSIBLE CONFIGURATIONS

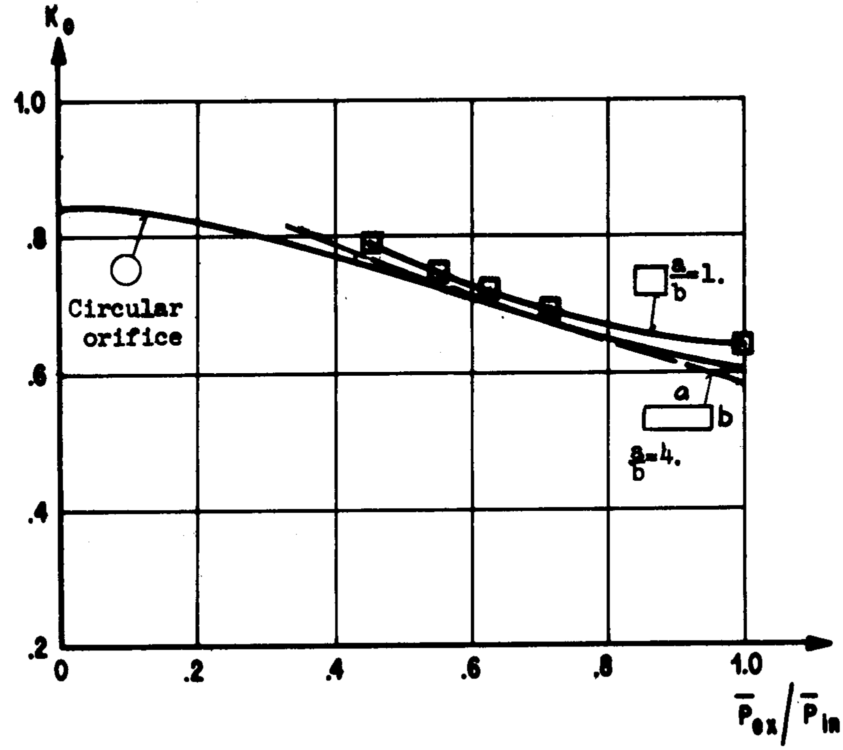


FIG. 2. DISCHARGE COEFFICIENT AS A FUNCTION OF COMPRESSIBILITY

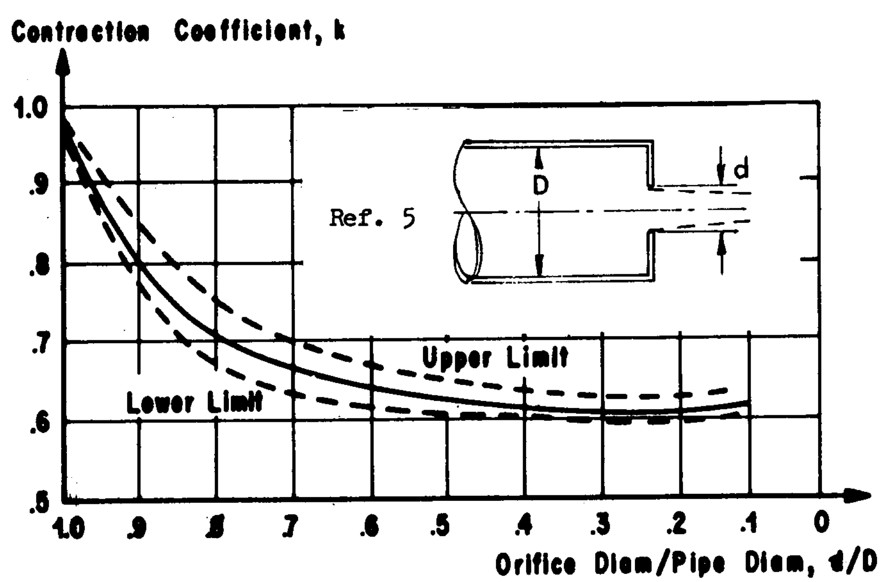


FIG. 3. DISCHARGE COEFFICIENT AS A FUNCTION OF ORIFICE-TO-PIPE DIAMETER

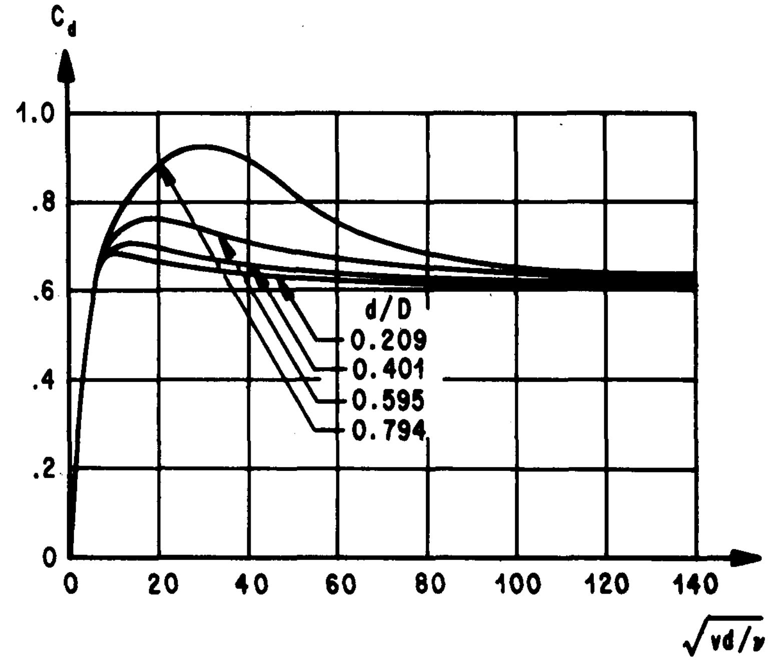


FIG. 4. DISCHARGE COEFFICIENTS FOR THE PLATE ORIFICE

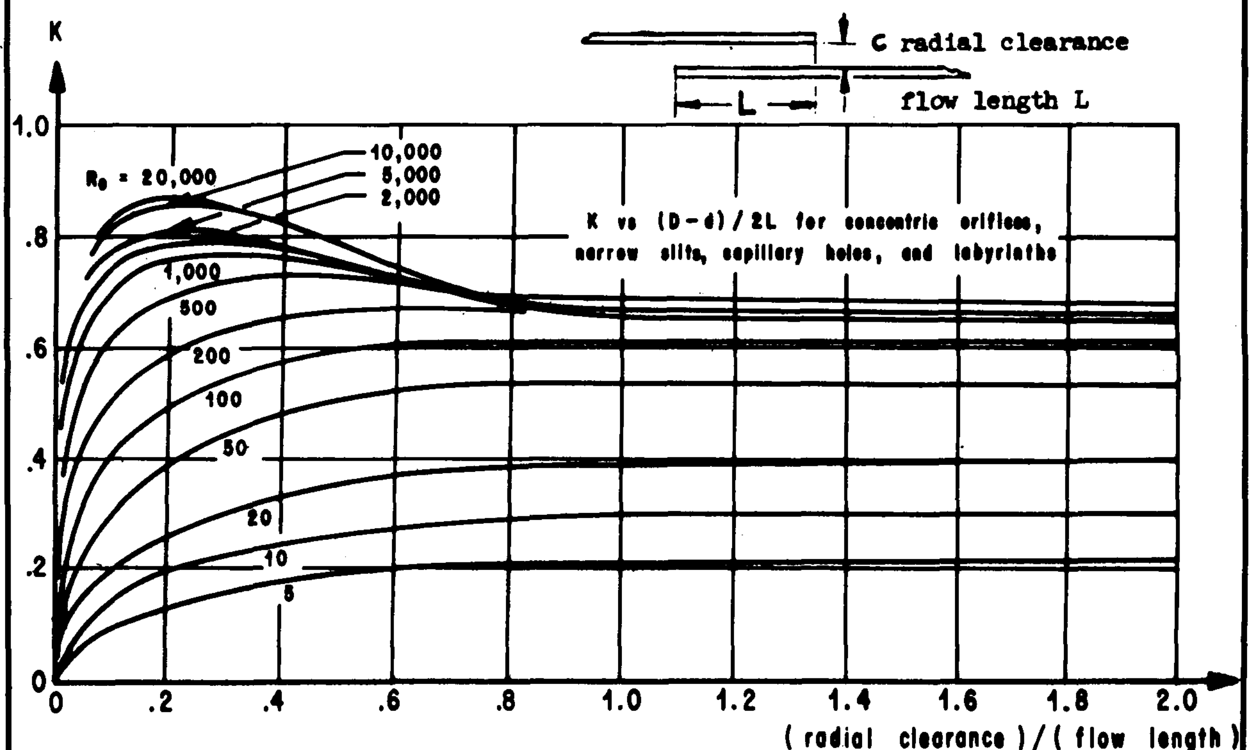


FIG. 5. LEAK DISCHARGE COEFFICIENT AS A FUNCTION OF REYNOLDS NUMBER AND RADI AL CLEARANCE TO FLOW LENGTH

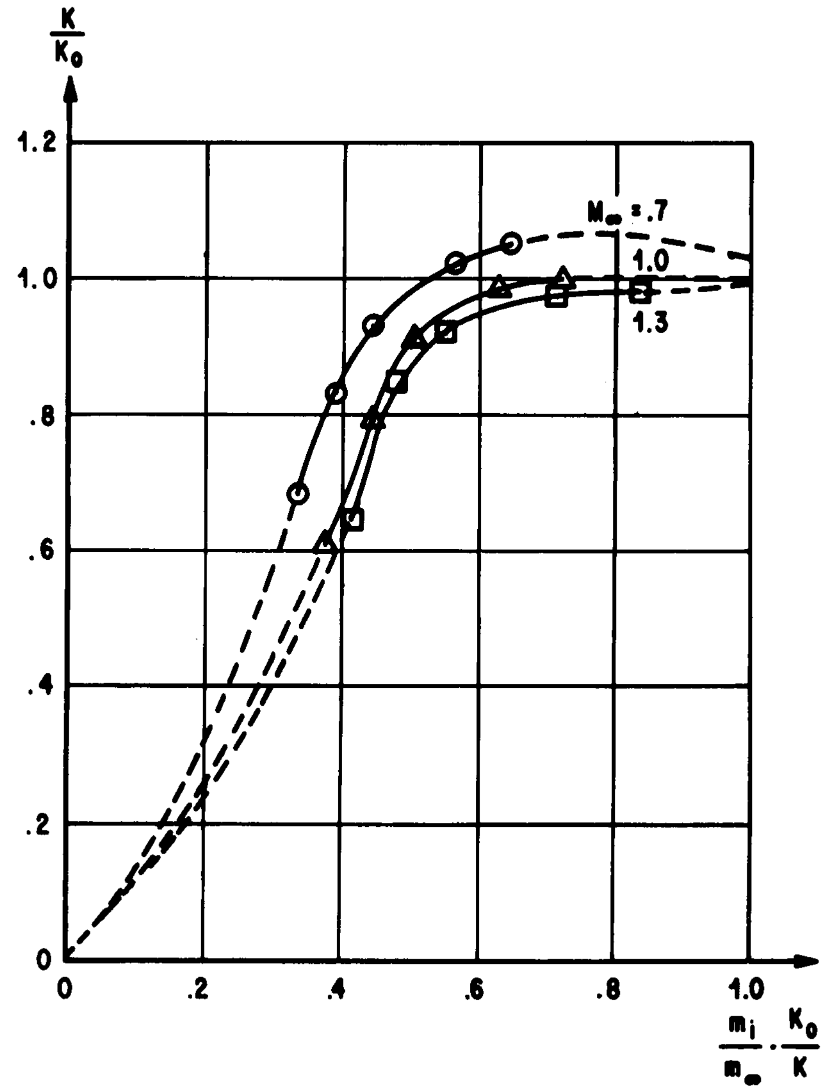


FIG. 6. VARIATION OF DISCHARGE COEFFICIENT WITH MASS FLOW RATIO

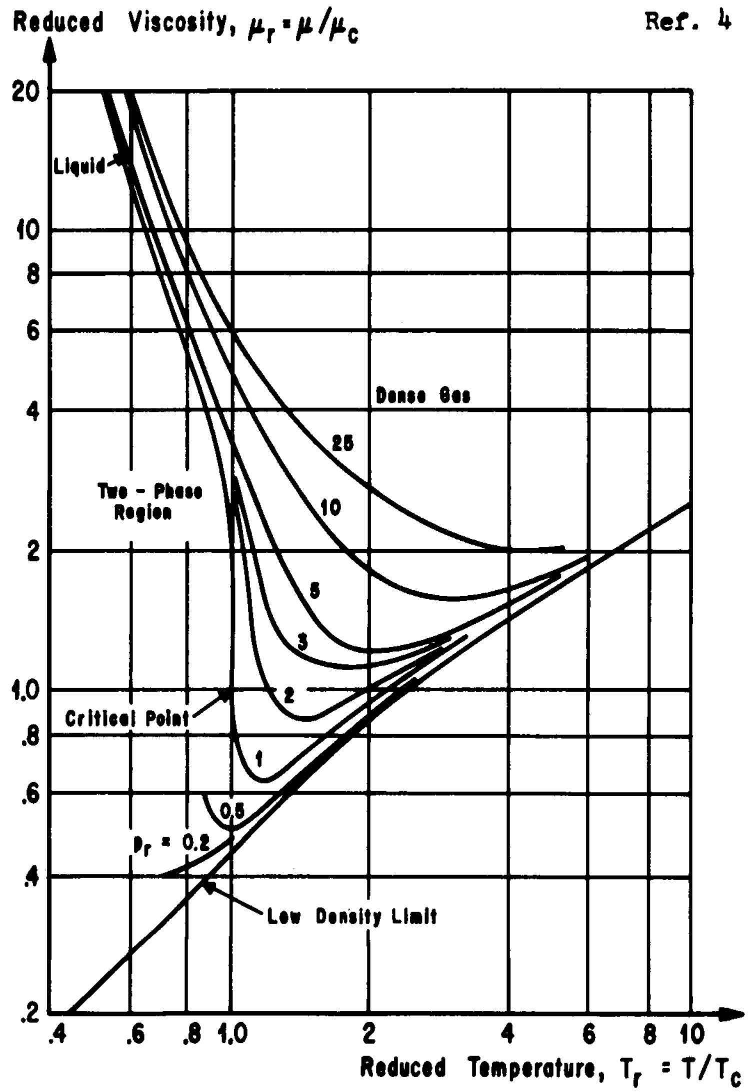
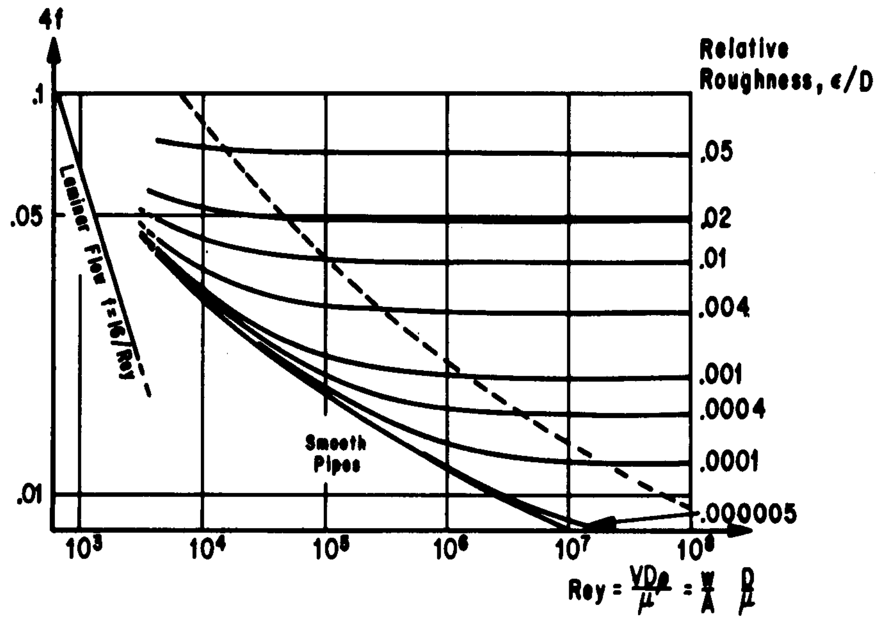


FIG. 7. REDUCED VISCOSITY ($\mu_r = \mu/\mu_c$)

AS A FUNCTION OF TEMPERATURE FOR SEVERAL VALUES OF THE REDUCED PRESSURE ($p_r = p/p_c$)



Roughness of pipe is measured by ϵ, and has typical values as follows (after Moody):

Pipe	ϵ, ft
Drawn tubing	.000005
Commercial steel	.00015
Asphalted cast iron	.0004
Galvanized iron	.0005
Cast iron	.00085
Concrete	.001-.01
Riveted steel	.003-.03

FIG. 8. FRICTION COEFFICIENT VERSUS PIPE REYNOLDS NUMBER FOR INCOMPRESSIBLE, FULLY DEVELOPED FLOW

TABLE 1

CRITICAL PROPERTIES

Substance	Molecular Weight M	Critical Constants[b,c,d] T_c (° K)	p_c (atm)	$\tilde{V}_c$ (cm³ g-mole⁻¹)	μ_c (g cm⁻¹ sec⁻¹) × 10⁶	k_c (cal sec⁻¹ cm⁻¹ ° K⁻¹) × 10⁶
Light elements:						
H_2	2.016	33.3	12.80	65.0	34.7	—
He	4.003	5.26	2.26	57.8	25.4	—
Noble gases:						
Ne	20.183	44.5	26.9	41.7	156.	79.2
Ar	39.944	151.	48.0	75.2	264.	71.0
Kr	83.80	209.4	54.3	92.2	396.	49.4
Xe	131.3	289.8	58.0	118.8	490.	40.2
Simple polyatomic substances:						
Air	28.97	132.	36.4	86.6	193.	90.8
N_2	28.02	126.2	33.5	90.1	180.	86.8
O_2	32.00	154.4	49.7	74.4	250.	105.3
O_3	48.00	268.	67.	89.4	—	—
CO	28.01	133.	34.5	93.1	190.	86.5
CO_2	44.01	304.2	72.9	94.0	343.	122.
NO	30.01	180.	64.	57.	258.	118.2
N_2O	44.02	309.7	71.7	96.3	332.	131.
SO_2	64.07	430.7	77.8	122.	411.	98.6
F_2	38.00	—	—	—	—	—
Cl_2	70.91	417.	76.1	124.	420.	97.0
Br_2	159.83	584.	102.	144.	—	—
I_2	253.82	800.	—	—	—	—
Hydrocarbons:						
CH_4	16.04	190.7	45.8	99.3	159.	158.0
C_2H_2	26.04	309.5	61.6	113.	237.	—
C_2H_4	28.05	282.4	50.0	124.	215.	—
C_2H_6	30.07	305.4	48.2	148.	210.	203.0
C_3H_6	42.08	365.0	45.5	181.	233.	—
C_3H_8	44.09	370.0	42.0	200.	228.	—
n-C_4H_{10}	58.12	425.2	37.5	255.	239.	—
i-C_4H_{10}	58.12	408.1	36.0	263.	239.	—
n-C_5H_{12}	72.15	469.8	33.3	311.	238.	—
n-C_6H_{14}	86.17	507.9	29.9	368.	248.	—
n-C_7H_{16}	100.20	540.2	27.0	426.	254.	—
n-C_8H_{18}	114.22	569.4	24.6	485.	259.	—
n-C_9H_{20}	128.25	595.0	22.5	543.	265.	—
Cyclohexane	84.16	553.	40.0	308.	284.	—
C_6H_6	78.11	562.6	48.6	260.	312.	—
Other organic compounds:						
CH_4	16.04	190.7	45.8	99.3	159.	158.0
CH_3Cl	50.49	416.3	65.9	143.	338.	—
CH_2Cl_2	84.94	510.	60.	—	—	—
$CHCl_3$	119.39	536.6	54.	240.	410.	—
CCl_4	153.84	556.4	45.0	276.	413.	—
C_2N_2	52.04	400.	59.	—	—	—
COS	60.08	378.	61.	—	—	—
CS_2	76.14	552.	78.	170.	404.	—

APPENDIX

PREPARATION OF THE INPUT DATA FOR THE DIFFERENT VENTING PROGRAMS

A. Venting of Compartments in Series

The simple discharge of a number of compartments in series is calculated with this program. The single compartments are connected by only one orifice. The first compartment, which vents into the atmosphere, can have more than one discharging orifice. For all compartments the possibility of leak flow is provided, with the restriction, however that only one leak per compartment can be handled.

All input data are read in by the INPUT subroutine. Floating-point numbers have the Format E15.8 and fixed-point numbers have I3. The data in consecutive order have the following meaning and dimensions.

NH, NP, MPRNT, NPO

NH = Fixed-point number, designating the number of trajectory data cards.

NP = Fixed-point number, designating the number of cards for the pressure data outside of the leaks of the compartments.

MPRNT = Fixed-point number for printing out check data. MPRNT greater than zero for print out.

NPO = Number of cards for the pressure data outside the orifices of the first compartment. The first compartment vents into the atmosphere.

OM, TC, PC, GC, G

OM = Molecular weight of the vented gas.

TC = Critical temperature of the vented gas in [°K].

PC = Critical pressure of the vented gas in atmospheres [Kg/cm^2].

GC = Gas constant of the vented gas [m/°K].

G = Specific heat ratio of vented gas.

TT(I), HT(I), FMT(I) NH times, maximum NH = 40

TT(I) = Trajectory time in seconds.

HT(I) = Trajectory altitude in meters m at time TT(I).

FMT(I) = Trajectory Mach number at time TT(I).

FM(J), CPF(J,1), CPF(J,2), ... NP times, maximum NP = 40

FMF(J) = Mach number for which the pressure coefficients CPF(J,I) are selected. It is not necessary that this Mach number correspond to the FMT(I) mentioned above. Both, however, must be in the same range. It is recommended that more points be selected in the transonic range.

CPF(J,I) = Pressure coefficients of the leaks, outside the compartment. If more compartments are to be vented, other cards must be added and the dimensions in the COMMON statements must be changed accordingly.

FMO(I), CPO(I,1), CPO(I,2), ... NPO times, maximum NPO = 40

FMO(I) = Mach number for which the orifice pressure coefficients COP(I,J) are selected.

CPO(I,J) = Orifice pressure coefficients of the first compartment outside the compartment.

NO, NT, N, NOR

NO = Fixed-point number, designating the first time step minus 1 second.

NT = Fixed-point number, designating the end time step minus 1 second.

N = Number of compartments in series also total number of leaks; one leak per compartment.

NOR = Fixed-point number; number of orifices of the first compartment venting into the atmosphere.

AO(I), G1(I), OM1(I) NOR times

AO(I) = Area of outside orifices of the first compartment $[m^2]$.

G1(I) = Mass flow through orifice number I of the first compartment [Kg sec/m].

OM1(I) = Mach number of orifice flow.

For the very first time step, insert a zero for G1(I) and OM1(I). After NT time steps, the OUTPUT subroutine will punch all cards beginning with the fixed-point numbers NO, NT, N, NOR and later.

A(I), VO(I), R1(I), P1(I), AM1(I)
AL(I), HR(I), CCF(I) N times

A(I) = Orifice area of the orifices connecting the compartments $[m^2]$.

VO(I) = Volume of the i-th compartment $[m^3]$.

R1(I) = Initial density of the gas in the compartments $[Kg\ sec^2/m^4]$.

P1(I) = Initial pressure of the gas in the compartments in $[kg/m^2]$.

AM1(I) = Initial mass flow out of the compartment [Kg sec/m].

AL(I) = Leak area of the compartments (representative mean value) $[m^2]$.

HR(I) = Hydraulic radius or mean radius of leaks defined in III.A $[m^2]$.

CCF(I) = Radial clearance over flow length of the leaks according to Figure 5.

The program can be restarted at any time step NT. The punched data replace the old corresponding data. The program punches for NT a fixed-point number. It can be changed to any other value NT ≥ NO.

B. The Input Data for the Venting Program with Pipe Flow

In this program, the pipe replaces the first compartment. Some of the input data have been changed. The format statements remain the same as in A. We repeat here also those symbols which have not changed.

NH, NP, MPRNT

Fixed-point numbers with the same meaning as in A.

OM, TC, PC, GC, G

Same meaning as in A.

TT(I), HT(I), FMT(I), QT(I)   NH times, maximum NH = 40

Same as in A. For the program with pipe flow and heat addition the term QT(I) is added.

QT(I) = Total heat flux per unit mass of gas [Kg m/sec].

FMF(I), CPF(I,1), CPF(I,2), ... NP times, maximum NP = 40

Same as in A.

NO, NT, N

Same as in A.

A(1), HR(1), PL(1), R1(1), P1(1)

A(1) = Cross section of the pipe [m^2].

HR(1) = Hydraulic radius of pipe [m].

PL(1) = Pipe length [m].

R1(1) = Initial density of the gas in the pipe [Kg sec^2/m^4].

P1(1) = Initial pressure of the gas in the pipe [Kg/m^2].

These data have been added to the original program.

A(I), VO(I), R1(I), P1(I), AM1(I)
AL(I), HR(I), CCF(I) N-1 times

Same as in A.

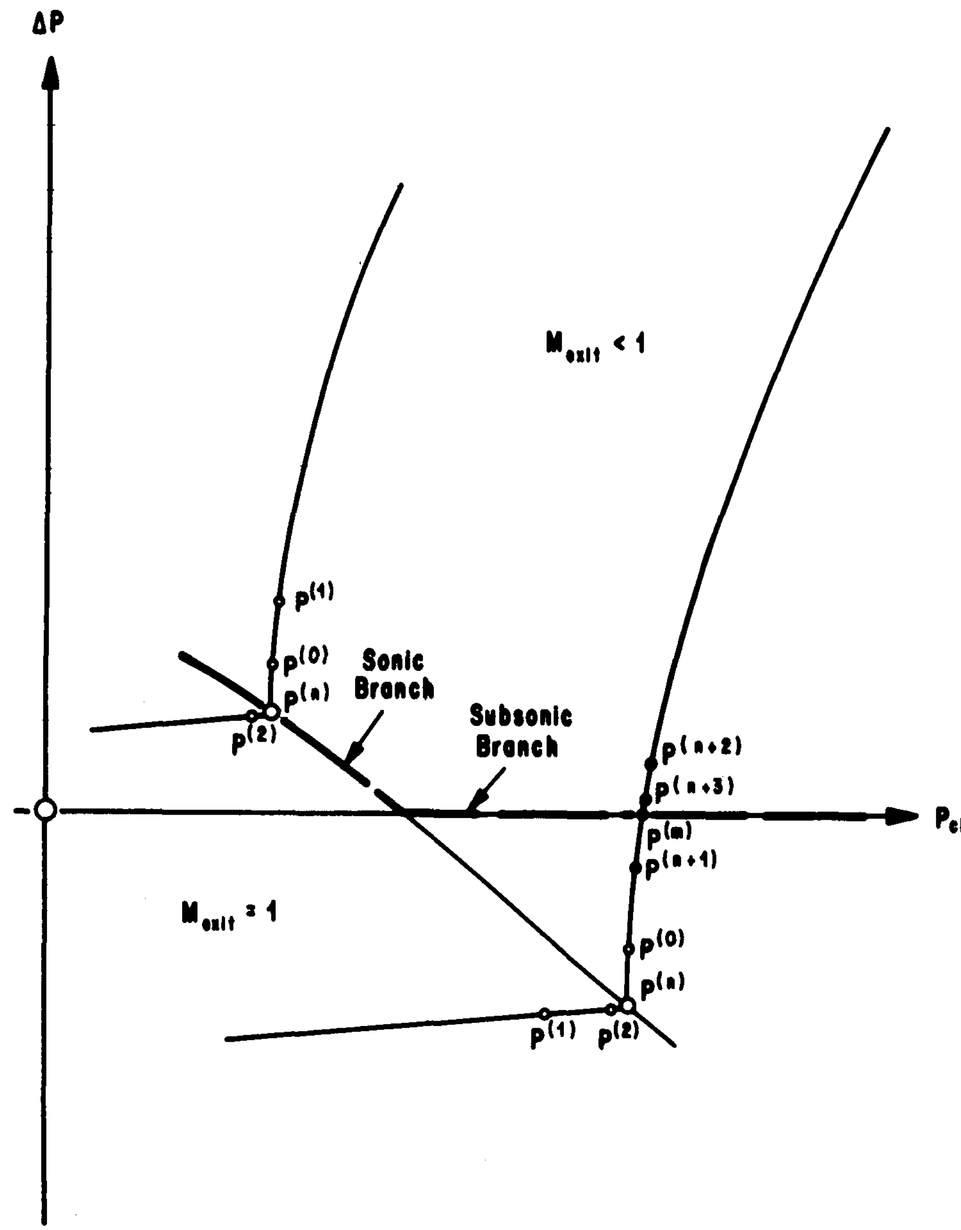
W

W = additional loss coefficient of the pipe as bends and elbows, etc. If no additional losses, enter zero.

C. Description of the Iteration Subroutine

This subroutine determines the next iterated pressure at the entrance of the pipe joining the upstream compartment. For subsonic exit Mach number the pressure at the exit has to match the external pressure. For this case the entrance pressure is varied according to a certain iteration scheme until the condition of equal pressures at the exit is met. For supersonic pipe flow the exit is choked and the pipe exit pressure is higher than the external pressure. Further expansion of the flow takes place external to the pipe.

In the subroutine the first trial value of the entrance pressure, which is determined in the main program by an extrapolation of entrance pressures occuring at an earlier time step. This pressure causes a certain exit pressure and exit Mach number, calculated by the subprogram PIPE. This program determines also an entrance Mach number which corresponds to a choked exit. The pressure difference across the exit $\Delta P = P_{exit} - P_{exit\,\infty}$ and the Mach number difference at the entrance $\Delta M = M_{entr} - M_{entr}(M_{ex} = 1)$ is listed according to whether M_{ex} is smaller than one (OK = ΔP) or equal to one (OG = ΔP). To obtain the second value of $P^{(1)}_{entr}$, the first one is perturbated by a fraction of the pressure difference $(P_{ch} - P^{(o)}_{entr})$, schematic A1. ΔP and ΔM is again listed. The next improved value is obtained by extrapolating the first and second approximation to make ΔM at the entrance of the pipe equal to zero. If ΔM has reached the tolerance, a logical decision is made. For $\Delta M \leqq$ Tol and $\Delta P < 0$ the pipe exit is not choked and the condition $\Delta P = 0$ must be sought. This condition is met by extrapolating PK (for which $M_{ex} < 1$) to OK = $\Delta P = 0$. If, however, $M \leqq$ Tol and $P \geqq 0$, the condition of choked exit flow is met within the tolerance.



SCHEMATIC A-1. SOLUTION FOR THE TWO DIFFERENT PIPE FLOW REGIMES ($M_{exit} \leq 1$)

```
      PROGRAM VENT
C     VENTING OF COMPARTMENTS IN SERIES
C     ****************************************
C     NOR =NUMBER OF ORIFICE OF THE FIRST COMPARTMENT
C     NPO = NUMBER OF CARDS OF PRESSURE DATA OUTSIDE OF ORIFICE
C     NH = NUMBER OF CARDS FOR TRAJECTORY DATA
C     N  =NUMBER OF COMPARTMENTS,ALSO NUMBER OF LEAKS
C         ONE LEAK PER COMPARTMENT
C         PIPE IS CONSIDERED AS COMPARTMENT NO. 1
C    HT = ALTITUDE AT TIME TT IN METERS
C    FMT = MACH NUMBER AT TIME TT
C     A  = ORIFICE AREA BETWEEN N COMP.IN METERS
C     VO = VOLUME OF COMPARTMENTS IN METERS3
C     CPF= PRESSURE COEFFICIENT AT MACH N.
C     A  = AREA OF NV ORIFICES OF COMP. 1 IN METERS
C     P  = PRESSURE IN COMPARTMENT IN (KG/M2)
C     CM1= COMPARTMENT MACH NUMBER
C     AL1= LEAK MASS FLOW
C     R  = DENSITY OF N COMP. IN KG.SEC./M4
C     AL =LEAK AREA OF COMP.N IN METERS
C     HR =HYDRAULIC RADIUS OR MEAN RADIUS IN METER
C     TC = CRITICAL TEMPERATURE OF GAS
C     GC = GAS CONSTANT OF GAS (M/DEG.K)
C     PC = CRITICAL PRESSURE OF GAS IN ATM(KG/CM2)
C     OM = MOL WEIGHT OF PARTICULAR GAS
C     CCF= RADIAL CLEARANCE OVER FLOW LENGTH
C     WI =ADDITIONAL LOSS COEFFICIENT FOR PIPE, IF NONE ENTER O.
C         IN PROPER FORMAT
C         NUMBER 1 COMP. IS THE COMP. THAT VENTS INTO THE ATMOSPHERE
C     AM1 = MASS FLOW OUT OF THE COMPARTMENT
C     G1  = MASS FLOW OUT OF ORIFICES OF THE FIRST COMPARTMENT
C     PG  = PRESSURE OUTSIDE OF THE ORIFICES OF THE FIRST COMPARTMENT
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM,WH(40,5)
      COMMON TP(40),OP(40),WM(40,5)
      DO 5 I=1,5
      AM1(I)=0.0
    5 AM2(I)=1.0E-07
      CALL INPUT
      D=(2./(G+1.))**(G/(G-1.))
      DO 13 I=NO,NT
      I1=I-1
      T1=I1
      T2=T1-1.0
      CALL TRAJ
      IF(I-1)4,11,4
```

```
    4  DO 15 L=1,40
       IF(MPRNT-4)24,24,25
   25  WRITE(61,555)
       WRITE(61,90)L,MPRNT
   24  IF(L-1)6,6,7
    6  DO 8 K=1,N
    8  AM1(K)=AM2(K)
    7  DO 20 K=1,N
       IF(K-1)1,1,2
    1  CONTINUE
       CALL PARTB(I,K,L,D)
       GO TO 23
    2  CONTINUE
       CALL PARTA(I,K,L,D)
   23  WH(L,K)=P1(K-1)
   20  WM(L,K)=AM1(K)
       IF(L-1)10,10,9
    9  DO 3 K=1,N
       IF(MPRNT)18,18,19
   19  WRITE(61,91)K,WM(L,K),WM(L-1,K)
   18  IF(ABS(WM(L,K)-WM(L-1,K))-1.0E-07)3,3,10
    3  CONTINUE
       GO TO 12
   10  IF(MPRNT)21,21,22
   22  WRITE(61,90)L,L,AM1(1),AM1(2),AM1(3),AM1(4)
   21  IF(L-3)15,16,16
   16  DO 17 K=3,N
   17  P1(K-1)=WH(L,K)-(WM(L,K)-WM(L-1,K))*(WH(L,K)-WH(L-1,K))/(WM(L,K)-
      12.0*WM(L-1,K)+WM(L-2,K))
   15  CONTINUE
       GO TO 12
   11  DO 14 IP=1,N
       CM1(IP)=+0.0
   14  AL1(IP)=+0.0
   12  CALL OUTPUT(I)
       WRITE(61,555)
   13  CONTINUE
       STOP
  555  FORMAT(1H0)
   90  FORMAT(2I4,8E15.8)
   91  FORMAT(I4,5E15.8)
       END
```

```
      SUBROUTINE PARTA(I,K,L,D)
C    VENTING OF COMPARTMENTS IN SERIES
      DIMENSION T(40),O(40)
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM,WH(40,5)
      T(1)=P2(K)*.999
      K1=K+1
      DO 30 M=1,40
      A1=(P1(K-1)+P2(K-1))/2.0
      R1(K)=R2(K)*((T(M)/P2(K))**(1./G))
      CALL APPT(K,A1,R1(K),T(M),A (K),A4,D)
      IF(AL(K))27,27,33
   27 AL1(K)=0.0
      GO TO 28
   33 Z1=PA/T(M)
      CALL LEAK(Z1,T(M),R1(K),AL(K),D,HR(K),CCF(K),AL1(K))
   28 S1=R2(K)*VO(K)
      S2=(2.0*(T(M)/P2(K))-1.0)**(1./G)
      S3=S1*(1.0-S2)+AM1(K1)-AL1(K)
      CMX2=2.0*((T(M)/A1)**((G-1.)/G)-1.0)/(G-1.)
      CM1(K)=SQRT(ABS(CMX2))
      IF(CM1(K)-1.0)6,6,7
    7 CM1(K)=1.0
    6 O(M)=A4-S3
      IF(M-1)54,54,35
   54 T(M+1)=T(M)*1.005
      GO TO 62
   35 T(M+1)=T(M)-O(M)*(T(M)-T(M-1))/(O(M)-O(M-1))
   62 P1(K)=2.0*T(M+1)-P2(K)
      AM1(K)=S3
      IF(ABS (O(M))-1.0E-07)22,30,30
   30 CONTINUE
      WRITE(61,99)
   22 IF(MPRNT)4,4,5
    5 WRITE(61,80)I,K,L,M,D,T(M),P1(K),S3,O(M),R1(K)
    4 RETURN
   80 FORMAT(4I4,6E15.8)
   99 FORMAT(17H  PARTA TOLERANCE)
      END
```

```
      SUBROUTINE PARTB(I,K,L,D)
C     VENTING OF COMPARTMENTS IN SERIES
      DIMENSION T(40),O(40),DO(40),TO(40)
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM,WH(40,5)
      COMMON TP(40),OP(40),WM(40,5)
      T(1)=P2(K)*.999
      K1=K+1
      DO 30 M=1,40
      R1(K)=R2(K)*((T(M)/P2(K))**(1./G))
      SUM=.0
      DO 6 J=1,NOR
      CALL APPT(J,PG(J),R1(K),T(M),AO(J),A4,D)
      A1=PG(J)/T(M)
      CALL COEF(A1,AK)
      AOM=RA*FM*CA
      Y=AK
      DO 3 J1=1,30
      TO(J1)=Y
      IF(A1-D)1,1,2
    1 A1=D
    2 RI=R1(K)*(A1**(1./G))
      AI=SQRT(ABS(G*T(M)*R1(K)))
      VI=AI*SQRT(ABS(2.*(1.-A1**((G-1.)/G))))
      AIM=RI*VI*AK
      X=AIM/(AOM*Y)
      CALL VENTC(X,AK,Y1)
      DO(J1)=Y1-Y
      IF(J1-2)8,8,9
    8 Y=.95*AK
      GO TO 12
    9 Y=TO(J1)-DO(J1)*(TO(J1)-TO(J1-1))/(DO(J1)-DO(J1-1))
   12 IF(MPRNT-4)10,10,13
   13 WRITE(61,81)J,J1,TO(J1),Y,DO(J1),Y1,A1,D,X
   10 IF(ABS(DO(J1))-1.0E-08)11,11,3
    3 CONTINUE
   11 G1(J)=A4*Y/AK
      SUM=SUM+G1(J)
      CMX2=2.0*((T(M)/PG(J))**((G-1.)/G)-1.0)/(G-1.)
      OM1(J)=SQRT(ABS(CMX2))
      IF(MPRNT-3)14,14,15
   15 WRITE(61,80)J,M,K,J1,A1,AK,A4,OM1(J),G1(J),T(M)
   14 IF(OM1(J)-1.0)6,6,7
    7 OM1(J)=1.0
    6 CONTINUE
      IF(AL(K))27,27,33
```

```
27   AL1(K)=0.0
     GO TO 28
33   Z1=PA/T(M)
     CALL LEAK(Z1,T(M),R1(K),AL(K),D,HR(K),CCF(K),AL1(K))
28   S1=R2(K)*VO(K)
     S2=(2.0*(T(M)/P2(K))-1.0)**(1./G)
     S3=S1*(1.0-S2)+AM1(K1)-AL1(K)
     O(M)=SUM-S3
     IF(M-1)54,54,35
54   T(M+1)=T(M)*1.005
     GO TO 62
35   T(M+1)=T(M)-O(M)*(T(M)-T(M-1))/(O(M)-O(M-1))
62   P1(K)=2.0*T(M+1)-P2(K)
     AM1(K)=S3
     IF(ABS (O(M))-1.0E-07)22,30,30
 30  CONTINUE
     WRITE(61,99)
22   IF(MPRNT)4,4,5
 5   WRITE(61,80)I,K,L,M,T(M),P1(K),S3,O(M),R1(K),SUM
 4   RETURN
80   FORMAT(4I4,7E15.8)
 99  FORMAT(17H  PARTA TOLERANCE)
81   FORMAT(2I4,8E15.8)
     END
```

```
      SUBROUTINE TRAJ
C     PA=AMBIENT PRESSURE
C     TA=TEMPERATURE
C     RA=DENSITY
C     CA=SPEED OF SOUND
C     PA1(N)=PRESSURE OUTSIDE PIPE AND LEAKS AT TIME T1
      DIMENSION W1(40)
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM
      CALL INTER(1,NH,1,1,TT,HT,T1,ALT)
      CALL INTER(1,NH,1,1,TT,FMT,T1,FM)
      DO 1 I=1,N
      DO 2 J=1,NP
    2 W1(J)=CPF(J,I)
      CALL INTER(1,NP,1,1,FMF,W1,FM,PA1(I))
    1 CONTINUE
      HG=ALT*6.378178E+06/(6.378178E+06+ALT)
      CALL ATMOS(HG,PA,TA,RA,CA)
      DO 3 I=1,N
    3 PA1(I)=PA+RA*PA1(I)*((FM*CA)**2)/2.0
      DO 4 I=1,NOR
      DO 5 J=1,NPO
    5 W1(J)=CPO(J,I)
      CALL INTER(1,NPO,1,1,FMO,W1,FM,PG(I))
    4 CONTINUE
      DO 6 I=1,NOR
    6 PG(I)=PA+RA*PG(I)*((FM*CA)**2)/2.0
      RETURN
      END
```

```
      SUBROUTINE INPUT
C     VENTING OF COMPARTMENTS IN SERIES
C     NP   = NUMBER OF CARDS FOR PRESSURE DATA OUTSIDE THE LEAKS
C     NH   = NUMBER OF CARDS FOR TRAJECTORY DATA
C     NPO  = NUMBER OF CARDS FOR PRESSURE AT ORIFICE
C     FMO  = MACH NUMBER FOR PRESSURE DATA OUTSIDE THE ORIFICE
C     CPO  = PRESSURE COEFFICIENT OUTSIDE THE ORIFICE
C     AO   = ORIFICE AREA
C     PG   = PRESSURE OUTSIDETHE ORIFICE
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM,WHT(40,5)
      READ(60,100)NH,NP,MPRNT,NPO
      READ(60,101)OM,TC,PC,GC,G
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      WRITE(61,126)GC,OM,TC,PC
      WRITE(61,128)VC
      DO 1 I=1,NH
    1 READ(60,101) TT(I),HT(I),FMT(I)
      DO 2 J=1,NP
    2 READ(60,101)FMF(J),CPF(J,1),CPF(J,2),CPF(J,3),CPF(J,4)
      DO 3 I=1,NPO
    3 READ(60,101)FMO(I),CPO(I,1),CPO(I,2),CPO(I,3),CPO(I,4)
      READ(60,100)NO,NT,N,NOR
      WRITE(61,130)
      DO 4 I=1,NOR
      READ(60,101)AO(I),G1(I),OM1(I)
      OM2(I)=OM1(I)
      G2(I)=G1(I)
    4 WRITE(61,124)I,AO(I)
      WRITE(61,108)
      DO 5 I=1,N
      READ(60,101) A(I),VO(I),R1(I),P1(I),AM1(I)
      READ(60,101) AL(I),HR(I),CCF(I),AL1(I)
      R2(I)=R1(I)
      P2(I)=P1(I)
      AL2(I)=AL1(I)
      AM2(I)=AM1(I)
      WRITE(61,125)I, A(I),VO(I),R1(I),P1(I)
    5 WRITE(61,127) AL(I),HR(I),CCF(I)
      AM2(N+1)=.0
      AM1(N+1)=.0
      WRITE(61,108)
      RETURN
  124 FORMAT(12H ORIFICE NR.I4,6H AREA E15.8,5H (M2))
  130 FORMAT(31H ORIFICE DATA FIRST COMPARTMENT)
  101 FORMAT(5E15.8)
```

```
 100 FORMAT(9I3)
 128 FORMAT(15H CRIT.VISCOSITY,E13.6,6H KG/MS)
 126 FORMAT(11H GAS CONST.,E13.6,5H M/OK,7H MOL W.,E13.6,9H CR.TEMP.,E1
    13.6,3H OK,9H CR.PRES.,E13.6,4H ATM)
 127 FORMAT(22H                 LEAK AREA,E15.8,3H M2,10H HYDR.RAD.E13.6,2H
    1 M,16H RAD.CLEAR/FL.L.,E13.6)
 125 FORMAT(9H COMP.NR.,I3,10H ORIF.AREA,E15.8,3H M2,5H VOL.,E15.8,3H M
    13,8H DENSITY,E15.8,8H KGS2/M4,7H PRES. E15.8,6H KG/M2)
108  FORMAT(1H0)
     END
```

```
      SUBROUTINE OUTPUT(INT)
C     VENTING OF COMPARTMENTS IN SERIES
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5),G1(5),OM1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5),G2(5),OM2(5)
      COMMON PG(5),CPO(40,5),FMO(40),AO(5),PA,TA,RA,CA,ALT,FM,WH(40,5)
      COMMON TP(40),OP(40),WM(40,5)
      WRITE(61,50)T1
   50 FORMAT(11H TIME       E12.5)
      WRITE(61,60)ALT,FM,PA,TA,RA
   60 FORMAT(11H ALTITUDE  E12.5,11H MACH NR.  E12.5,11H AMB.PR.    E12.5
     1,11H AMB.TEMP  E12.5,11H AMB.DENS  E12.5)
      DPN=P1(1)-PA
      I=1
      WRITE(61,61)I,P1(1),R1(1),DPN,AM1(1)
   61 FORMAT(12H COMPARTM.  I3,19H         PRESSURE  E12.5,11H DENSITY
     1 E12.5,11H PRES.DIFF.E12.5,11H MASS FLOW E12.5)
      WRITE(61,65)PA1(1),AL1(1)
   65 FORMAT(34H                         LEAK PR.  E12.5,11H LEAK M.FL.E1
     12.5)
      DO 8 I=1,NOR
      PRR=P1(1)/PG(I)
    8 WRITE(61,66)I,PG(I),G1(I),OM1(I),PRR
   66 FORMAT(15H          ORIF.I3,16H      ORIF.PRES.E12.5,11H MASS FLOW
     1 E12.5,11H ORIF.M.NR.E12.5,11H PRES.RATIOE12.5)
      DO 1 I=2,N
      DPN=P1(I)-PA
      WRITE(61,63)I,P1(I),R1(I),CM1(I),AM1(I)
    1 WRITE(61,64)PA1(I),AL1(I),DPN
   63 FORMAT(12H COMPARTM.  I3,19H         PRESSURE  E12.5,11H DENSITY
     1 E12.5,11H MACH NR   E12.5,11H MASS FLOW E12.5)
   64 FORMAT(34H                         LEAK PR.  E12.5,11H LEAK M.FL.E1
     12.5,11H PRES.DIFF.E12.5)
      IF(INT-NT)4,6,6
    6 NO1=NT+1
      NT1=120
      WRITE(62,80)NO1,NT1,N,NOR
      DO 9 I=1,NOR
    9 WRITE(62,81)AO(I),G1(I),OM1(I)
      DO 2 I=1,N
      WRITE(62,81)A(I),VO(I),R1(I),P1(I),AM1(I)
    2 WRITE(62,81)AL(I),HR(I),CCF(I),AL1(I)
    4 DO 3 I=1,N
      AM2(I)=AM1(I)
      IF(INT-1)5,7,5
    7 AM2(I)=AM1(I)+1.0E-06
    5 P2(I)=P1(I)
      R2(I)=R1(I)
```

```
      CM2(I)=CM1(I)
 3    AL2(I)=AL1(I)
      AM2(N+1)=0.0
      AM1(N+1)=0.0
      P1(1)=.995*P1(1)
      DO 10 I=1,NOR
      G2(I)=G1(I)
10    OM2(I)=OM1(I)
      RETURN
80    FORMAT(9I3)
81    FORMAT(5E15.8)
      END
```

```
      SUBROUTINE APPT(K,PX,Z2,Z3,Z4,A4,D)
C     CALCULATES IF FLOW IS SONIC OR SUBSONIC IN COMPARTMENT APPT.
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G
      A1=PX/Z3
      CALL COEF(A1,AK)
      IF(A1-D)3,3,4
    4 A2=AK*SQRT (2.*Z3*Z2)*Z4
      A2=A2*(A1**(1./G))
      A3=A1**((G-1.)/G)
      IF(A3-1.)5,6,6
    5 A4=A2*SQRT (G*(1.-A3)/(G-1.))
      GO TO 20
    6 A4=-A2*SQRT (G*ABS (1.-A3)/(G-1.))
      GO TO 20
    3 A2=AK*Z4*SQRT (2.*Z3*Z2)
      A4=A2*((2./(G+1.))**(1./(G-1.)))*SQRT (G/(G+1.))
   20 RETURN
      END
```

```
      SUBROUTINE LEAK(Z1,A,B,C,D,E,RCF,A4)
C     CALCULATES IF FLOW IS SONIC OR SUBSONIC IN ORIFICE.
C  Z1   PRESSURE RATIO, EXTERN. TO INTERN. PRESSURE
C  A    INTERN.PRESSURE
C  B    INTERN.DENSITY
C  C    CLOSE CLEARENCE ORIFICE AREA
C  G    SPEC. HEAT RATIO
C  D    DISTING. VALUE FOR CHOKING
C  E    HYDRAUL. RADIUS
C  PC   CRITICAL PRESSURE OF GAS
C  TC   CRITICAL TEMP. OF GAS
C  R    GAS CONSTANT FOR PART. GAS.
C  OM   MOL WEIGHT OF GAS
C  A4   MASS FLOW THROUGH CLOSE-CLEAR ORIF.
      COMMON MPRNT,NOR,NPO
      COMMON NH,N,NT,NP,RG,OM,TC,PC,G
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      CALL COEF(Z1,AK)
      IF(Z1-D)2,2,1
    1 A2=AK*SQRT (2.*A*B)*C
      A2=A2*(Z1**(1./G))
      A3=Z1**((G-1.)/G)
      IF(A3-1.)3,4,4
    3 A4=A2*SQRT (G*(1.-A3)/(G-1.))
      L=-1
      GO TO 5
    4 A4=-A2*SQRT (G*ABS (1.-A3)/(G-1.))
      L=-1
      GO TO 5
    2 A2=AK*C*SQRT (2.*A*B)
      A4=A2*((2./(G+1.))**(1./(G-1.)))*SQRT (G/(G+1.))
      L=1
    5 T=A/(B*RG*9.78035)
      CALL VISC(T,TC,VC,AMU)
      RE=ABS (A4*E/(C*AMU))
      CALL CLEAR(RE,RCF,Y)
      A4=A4*Y
      CALL COEF(Z1,X)
      A4=A4*X
      RETURN
      END
```

```
      SUBROUTINE ATMOS(HG,P,T,R,C)
      AM=28.9644
      RS=847.8118
      A=6378178.
      G0=9.80665
      GA=1.4
      IF(HG-11000.)1,2,2
    1 PB=10332.3
      TP=-6.5/1000.
      TM=288.15
      HB=0.
      GO TO 10
   2  IF(HG-20000.)3,4,4
    3 PB=2307.83
      TP=0.
      TM=216.65
      HB=11000.
      GO TO 12
   4  IF(HG-32000.)5,6,6
    5 PB=558.283
      TP=1./1000.
      TM=216.65
      HB=20000.
      GO TO 10
   6  IF(HG-47000.)7,8,8
    7 PB=88.513
      TP=2.8/1000.
      TM=228.65
      HB=32000.
      GO TO 10
   8  IF(HG-52000.)9,13,13
    9 PB=11.309
      TM=270.65
      HB=47000.
      TP=0.
      GO TO 12
   13 PB=6.0164
      TM=270.65
      HB=52000.
      TP=-2./1000.
   10 A=ALOG(TM/(TM+TP*(HG-HB)))
      P=PB*EXP (AM*A/(RS*TP))
      GO TO 11
   12 P=PB*EXP (-AM*(HG-HB)/(RS*TM))
   11 T=TM+TP*(HG-HB)
      R=AM*P/(RS*T*G0)
      C=SQRT (GA*RS*G0*T/AM)
      RETURN
      END
```

```
      SUBROUTINE INTER(N,M,NN,MN,X,Y,XS,YS)
      DIMENSION X(50),Y(50),XS(140),YS(140),AL(5)
      DO 1 I=NN,MN
      DO 2 L=N,M
      IF(XS(I)-X(L))3,2,2
    2 CONTINUE
      L=M
    3 L1=L
      IF(L1-N-1)4,4,5
    4 L1=N
      GO TO 8
    5 IF(L1-M+1)6,6,7
    6 L1=L1-2
      GO TO 8
    7 L1=M-3
    8 L2=L1+3
      DO 9 J=L1,L2
      II=J-L1+1
      AL(II)=1.
      DO 9 LP=L1,L2
      IJ=LP-L1+1
      IF(II-IJ)11,9,11
   11 AL(II)=AL(II)*(XS(I)-X(LP))/(X(J)-X(LP))
    9 CONTINUE
      SUM =0.
      DO 10 IP=L1,L2
      II=IP-L1+1
   10 SUM=SUM+AL(II)*Y(IP)
    1 YS(I)=SUM
      RETURN
      END
```

```
      SUBROUTINE VISC(T,TC,VC,A)
      TR=T/TC
      IF(TR-1.5)1,1,2
    1 A1=.442
      AN=.9882
      GO TO 9
    2 IF(TR-2.)3,3,4
    3 A1=.462
      AN=.8204
      GO TO 9
    4 IF(TR-3.)5,5,6
    5 A1=.5
      AN=.6480
      GO TO 9
    6 IF(TR-5.)7,7,8
    7 A1=.564
      AN=.5185
      GO TO 9
    8 A1=.631
      AN=.5099
    9 A=(A1*TR)**AN
      A=A*VC
      RETURN
      END
```

```
      SUBROUTINE CLEAR(RE,RCF,Y)
      IF(RE-100.)1,1,2
    1 A=.02
      AN=.1867
      GO TO 5
    2 IF(RE-500.)3,3,4
    3 A=.52
      AN=.0782
      GO TO 5
    4 A=1.0
      AN=0.
    5 Y=(A+AN*ALOG(1.+RE))*(1.-EXP (-4.788*SQRT (RCF)))
      RETURN
      END
```

```
      SUBROUTINE COEF(A1,AK)
      CON=1.0
      AB=A1
      IF(AB-1.)1,1,2
2     AB=1./A1
      CON=-1.0
1     A=EXP (AB)
      B=1./A
      C=EXP (-3.5*AB)
      AK=(1.-.155*C-.5186*(A-B)/(A+B))*CON
      RETURN
      END
```

```
SUBROUTINE VENTC(X,AK,Y)
S=4.5714285
U=.0101
W=X**S
Q=W/((1.-X)*U+W)
Y=AK*Q
RETURN
END
```

```
      PROGRAM VENT
C   VENTING THROUGH A PIPE, NO HEAT ADDITION
C    *****************************************
C    NH = NUMBER OF CARDS FOR TRAJECTORY DATA
C    N  =NUMBER OF COMPARTMENTS,ALSO NUMBER OF LEAKS
C        ONE LEAK PER COMPARTMENT
C        PIPE IS CONSIDERED AS COMPARTMENT NO. 1
C   HT = ALTITUDE AT TIME TT IN METERS
C   FMT = MACH NUMBER AT TIME TT
C    A  = ORIFICE AREA BETWEEN N COMP.IN METERS
C    VO = VOLUME OF COMPARTMENTS IN METERS3
C    CPF= PRESSURE COEFFICIENT AT MACH N.
C    A  = AREA OF NV ORIFICES OF COMP. 1 IN METERS
C    P  = PRESSURE IN COMPARTMENT IN (KG/M2)
C    CM1= COMPARTMENT MACH NUMBER
C    AL1= LEAK MASS FLOW
C    R  = DENSITY OF N COMP. IN KG.SEC./M4
C    AL =LEAK AREA OF COMP.N IN METERS
C    HR =HYDRAULIC RADIUS OR MEAN RADIUS IN METER
C    TC = CRITICAL TEMPERATURE OF GAS
C    GC = GAS CONSTANT OF GAS (M/DEG.K)
C    PC = CRITICAL PRESSURE OF GAS IN ATM(KG/CM2)
C    OM = MOL WEIGHT OF PARTICULAR GAS
C    CCF= RADIAL CLEARANCE OVER FLOW LENGTH
C    WI =ADDITIONAL LOSS COEFFICIENT FOR PIPE, IF NONE ENTER 0.
C        IN PROPER FORMAT
C        NUMBER 1 COMP. IS THE COMP. THAT VENTS INTO THE ATMOSPHERE
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5)
      DO 5 I=1,5
      AM1(I)=0.0
    5 AM2(I)=1.0E-07
      CALL INPUT
      D=(2./(G+1.))**(G/(G-1.))
      DO 13 I=NO,NT
      I1=I-1
      T1=I1
      T2=T1-1.0
      CALL TRAJ(ALT,FM,HG,PA,TA,RA,CA)
      IF(I-1)4,11,4
    4 DO 15 L=1,40
      IF(L-1)6,6,7
    6 DO 8 K=1,N
    8 AM1(K)=AM2(K)
    7 DO 20 K=2,N
```

```
      IF(K-2)1,1,2
    1 CONTINUE
      CALL PARTB(K,L,I)
      GO TO 23
    2 CONTINUE
      CALL PARTA(I,K,L,D)
   23 WH(L,K)=P1(K-1)
   20 WM(L,K)=AM1(K)
      IF(L-1)10,10,9
    9 DO 3 K=1,N
      IF(MPRNT)18,18,19
   19 WRITE(61,91)K,WM(L,K),WM(L-1,K)
   18 IF(ABS(WM(L,K)-WM(L-1,K))-1.0E-07)3,3,10
    3 CONTINUE
      GO TO 12
   10 IF(MPRNT)21,21,22
   22 WRITE(61,90)L,L,AM1(1),AM1(2),AM1(3),AM1(4)
   21 IF(L-3)15,16,16
   16 DO 17 K=3,N
   17 P1(K-1)=WH(L,K)-(WM(L,K)-WM(L-1,K))*(WH(L,K)-WH(L-1,K))/(WM(L,K)-
     12.0*WM(L-1,K)+WM(L-2,K))
   15 CONTINUE
      GO TO 12
   11 EM=0.0
      EMX=0.0
      DO 14 IP=1,N
      CM1(IP)=+0.0
   14 AL1(IP)=+0.0
   12 CALL OUTPUT(I)
      WRITE(61,555)
   13 CONTINUE
      STOP
  555 FORMAT(1H0)
   90 FORMAT(2I4,8E15.8)
   91 FORMAT(I4,5E15.8)
      END
```

```
      SUBROUTINE PARTA(I,K,L,D)
      DIMENSION T(41),O(41)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      T(1)=(P2(K)+P1(K))/2.0
      K1=K+1
      DO 30 M=1,40
      A1=(P1(K-1)+P2(K-1))/2.0
      R1(K)=R2(K)*((T(M)/P2(K))**(1./G))
      CALL APPT(K,A1,R1(K),T(M),A (K),A4,D)
      IF(AL(K))27,27,33
   27 AL1(K)=0.0
      GO TO 28
   33 Z1=PA/T(M)
      CALL LEAK(Z1,T(M),R1(K),AL(K),D,HR(K),CCF(K),AL1(K))
   28 S1=R2(K)*VO(K)
      S2=(2.0*(T(M)/P2(K))-1.0)**(1./G)
      S3=S1*(1.0-S2)+AM1(K1)-AL1(K)
      CMX2=2.0*((T(M)/A1)**((G-1.)/G)-1.0)/(G-1.)
      CM1(K)=SQRT(ABS(CMX2))
      IF(CM1(K)-1.0)6,6,7
    7 CM1(K)=1.0
    6 O(M)=A4-S3
      IF(M-1)54,54,35
   54 T(M+1)=T(M)*1.005
      GO TO 62
   35 T(M+1)=T(M)-O(M)*(T(M)-T(M-1))/(O(M)-O(M-1))
   62 P1(K)=2.0*T(M+1)-P2(K)
      AM1(K)=S3
      IF(MPRNT-2)1,1,3
    3 WRITE(61,90)M,T(M),A1,A4,A(K),AL1(K),S2,S3,T(M+1)
   90 FORMAT(I3,8E15.8)
    1 IF(ABS (O(M))-1.0E-07)22,30,30
   30 CONTINUE
      WRITE(61,99)
   22 IF(MPRNT)4,4,5
    5 WRITE(61,80)I,K,L,M,D,T(M),P1(K),S3,O(M),R1(K)
    4 RETURN
   80 FORMAT(4I4,6E15.8)
   99 FORMAT(17H  PARTA TOLERANCE)
      END
```

```
      SUBROUTINE PARTB(K1,L1,I)
      DIMENSION PM(41)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5)
      K=K1-1
      NU=0
      NS=0
      DO 1 L=1,40
      IF(MPRNT-1)2,2,10
 10   WRITE(61,555)
      WRITE(61,100)L,K
  2   IF(L-2)6,3,5
  6   PM(L)=P1(K1)
      GO TO 5
  3   PM(L)=.998*PM(L-1)
  5   P1(K1)=PM(L)
      TL=P1(K1)/(R1(K1)*GC*9.8)
      CALL ENTR(K,SV)
      PM(L)=P1(K1)
      AM1(K1)=AM1(K)
      HPO=EM
      TL=TL/(1.+(G-1.)*EM *EM /2.)
      CALL FRICT(AM1(K ),A(K),TL,FR,VIS,RN)
      FF=G*(FR*PL(K)/(2.*HR(K))+W)
      CALL PIPE(FF,CE,SV,HP,GLIM)
      IF(MPRNT-1)11,11,12
 12   WRITE(61,100)L,K,L1,P1(K),EP,EM,EMX,PA1(K),AM1(K),AM1(K1),HP
      WRITE(61,100)L,K1,L1,P1(K1),R1(K1),P2(K1),P2(K)
 11   DE=P1(K)-PA1(K)
      DEM=HPO-HP
      CALL ITER(NU,NS,PM(L),DE,DEM,PM(L+1),MSB,MSO)
      IF(MPRNT-1)13,13,14
 14   WRITE(61,101)L,K,AM1(K1),PM(L),PM(L+1),DE,DEM
 13   IF(EMX-1.)17,7,7
 17   IF(MSB)4,8,4
  4   IF(ABS(DE)-(1.0E-08*P1(K1)))8,8,1
  7   IF(DE+(2.0E-08*P1(K1)))1,9,9
  9   IF(ABS(DEM)-1.0E-07)8,8,18
 18   IF(MSO)1,8,1
  1   CONTINUE
  8   WM(L1,K)=AM1(K)
      WM(L1,K1)=AM1(K1)
      CM1(K1)=EM
      CM1(K)=EMX
      IF(MPRNT)15,15,16
```

```
 16   WRITE(61,102)L1,L,MSB,MSO,AM1(K),P1(K1),P2(K1)
 15   RETURN
101   FORMAT(2I4,6E15.8)
100   FORMAT(3I4,8E15.8)
102   FORMAT(4I4,8E15.8)
555   FORMAT(1H0)
      END
```

```
      SUBROUTINE INPUT
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      READ(60,100)NH,NP,MPRNT
      READ(60,101)OM,TC,PC,GC,G
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      WRITE(61,126)GC,OM,TC,PC
      WRITE(61,128)VC
      DO 1 I=1,NH
1     READ(60,101) TT(I),HT(I),FMT(I)
      DO 2 J=1,NP
2     READ(60,101)FMF(J),CPF(J,1),CPF(J,2),CPF(J,3),CPF(J,4)
      READ(60,100)NO,NT,N
      WRITE(61,130)
      READ(60,101) A(1),HR(1),PL(1),R1(1),P1(1)
      WRITE(61,124)A(1),HR(1),PL(1)
      WRITE(61,108)
      DO 5 I=2,N
      READ(60,101) A(I),VO(I),R1(I),P1(I),AM1(I)
      READ(60,101) AL(I),HR(I),CCF(I),AL1(I)
      R2(I)=R1(I)
      P2(I)=P1(I)
      AL2(I)=AL1(I)
      AM2(I)=AM1(I)
      WRITE(61,125)I, A(I),VO(I),R1(I),P1(I)
5     WRITE(61,127) AL(I),HR(I),CCF(I)
      AM2(1)=AM1(2)
      EP=P1(2)
      ER=R1(2)
      READ(60,101)W
      WRITE(61,108)
      RETURN
124   FORMAT(10H PIPE AREA,E15.8,10H HYDR.RAD.,E15.8,8H PIPE L.,E15.8)
130   FORMAT(13H PIPE SECTION)
101   FORMAT(5E15.8)
100   FORMAT(9I3)
128   FORMAT(15H CRIT.VISCOSITY,E13.6,6H KG/MS)
126   FORMAT(11H GAS CONST.,E13.6,5H M/OK,7H MOL W.,E13.6,9H CR.TEMP.,E1
     13.6,3H OK,9H CR.PRES.,E13.6,4H ATM)
127   FORMAT(22H                    LEAK AREA,E15.8,3H M2,10H HYDR.RAD.E13.6,2H
     1 M,16H RAD.CLEAR/FL.L.,E13.6)
125   FORMAT(9H COMP.NR.,I3,10H ORIF.AREA,E15.8,3H M2,5H VOL.,E15.8,3H M
     13,8H DENSITY,E15.8,8H KGS2/M4,7H PRES. E15.8,6H KG/M2)
108   FORMAT(1H0)
      END
```

```
      SUBROUTINE OUTPUT(INT)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5)
      WRITE(61,50)T1
   50 FORMAT(11H TIME       E12.5)
      WRITE(61,60)ALT,FM,PA,TA,RA
   60 FORMAT(11H ALTITUDE  E12.5,11H MACH NR.  E12.5,11H AMB.PR.   E12.5
     1,11H AMB.TEMP  E12.5,11H AMB.DENS  E12.5)
      WRITE(61,61)EP,ER,EM,AM1(1)
   61 FORMAT(34H COMPARTM.    PIPE       ENTR.PR.  E12.5,11H ENTR.DENS.E1
     12.5,11H ENTR.M.NR.E12.5,11H ENTR.M.FL.E12.5)
      WRITE(61,62)P1(1),R1(1),EMX,AM1(1)
   62 FORMAT(34H                         EXIT PR.  E12.5,11H EXIT.DENS E1
     12.5,11H EXIT M.NR.E12.5,11H EXIT M.FL.E12.5)
      DO 1 I=2,N
      DPN=P1(I)-PA
      WRITE(61,63)I,P1(I),R1(I),CM1(I),AM1(I)
    1 WRITE(61,64)PA1(I),AL1(I),DPN
   63 FORMAT(12H COMPARTM.  I3,19H          PRESSURE  E12.5,11H DENSITY
     1 E12.5,11H MACH NR   E12.5,11H MASS FLOW E12.5)
   64 FORMAT(34H                         LEAK PR.  E12.5,11H LEAK M.FL.E1
     12.5,11H PRES.DIFF.E12.5)
      IF(INT-NT)4,6,6
    6 NO1=NT+1
      NT1=120
      WRITE(62,80)NO1,NT1,N
      WRITE(62,81)A(1),HR(1),PL(1),R1(1),P1(1)
      DO 2 I=2,N
      WRITE(62,81)A(I),VO(I),R1(I),P1(I),AM1(I)
    2 WRITE(62,81)AL(I),HR(I),CCF(I),AL1(I)
      WRITE(62,81)W
    4 DO 3 I=1,N
      AM2(I)=AM1(I)
      IF(INT-1)5,7,5
    7 AM2(I)=AM1(I)+1.0E-06
    5 P2(I)=P1(I)
      R2(I)=R1(I)
      CM2(I)=CM1(I)
    3 AL2(I)=AL1(I)
      AM2(N+1)=0.0
      AM1(N+1)=0.0
      EP=.99*EP
      RETURN
   80 FORMAT(9I3)
   81 FORMAT(5E15.8)
      END
```

```
      SUBROUTINE APPT(K,PX,Z2,Z3,Z4,A4,D)
C     CALCULATES IF FLOW IS SONIC OR SUBSONIC IN COMPARTMENT APPT.
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G
      A1=PX/Z3
      CALL COEF(A1,AK)
      IF(A1-D)3,3,4
    4 A2=AK*SQRT (2.*Z3*Z2)*Z4
      A2=A2*(A1**(1./G))
      A3=A1**((G-1.)/G)
      IF(A3-1.)5,6,6
    5 A4=A2*SQRT (G*(1.-A3)/(G-1.))
      GO TO 20
    6 A4=-A2*SQRT (G*ABS (1.-A3)/(G-1.))
      GO TO 20
    3 A2=AK*Z4*SQRT (2.*Z3*Z2)
      A4=A2*((2./(G+1.))**(1./(G-1.)))*SQRT (G/(G+1.))
   20 IF(A1-1.)1,1,2
    2 IF(A4)1,1,8
    8 A4=-A4
    1 RETURN
      END
```

```
      SUBROUTINE LEAK(Z1,A,B,C,D,E,RCF,A4)
C     CALCULATES IF FLOW IS SONIC OR SUBSONIC IN ORIFICE.
C  Z1   PRESSURE RATIO, EXTERN. TO INTERN. PRESSURE
C  A    INTERN.PRESSURE
C  B    INTERN.DENSITY
C  C    CLOSE CLEARENCE ORIFICE AREA
C  G    SPEC. HEAT RATIO
C  D    DISTING. VALUE FOR CHOKING
C  E    HYDRAUL. RADIUS
C  PC   CRITICAL PRESSURE OF GAS
C  TC   CRITICAL TEMP. OF GAS
C  R    GAS CONSTANT FOR PART. GAS
C  OM   MOL WEIGHT OF GAS
C  A4   MASS FLOW THROUGH CLOSE-CLEAR ORIF.
      COMMON MPRNT
      COMMON NH,N,NT,NP,RG,OM,TC,PC,G
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      CALL COEF(Z1,AK)
      IF(Z1-D)2,2,1
    1 A2=AK*SQRT (2.*A*B)*C
      A2=A2*(Z1**(1./G))
      A3=Z1**((G-1.)/G)
      IF(A3-1.)3,4,4
    3 A4=A2*SQRT (G*(1.-A3)/(G-1.))
      L=-1
      GO TO 5
    4 A4=-A2*SQRT (G*ABS (1.-A3)/(G-1.))
      L=-1
      GO TO 5
    2 A2=AK*C*SQRT (2.*A*B)
      A4=A2*((2./(G+1.))**(1./(G-1.)))*SQRT (G/(G+1.))
      L=1
    5 T=A/(B*RG*9.78035)
      CALL VISC(T,TC,VC,AMU)
      RE=ABS (A4*E/(C*AMU))
      CALL CLEAR(RE,RCF,Y)
      A4=A4*Y
      CALL COEF(Z1,X)
      A4=A4*X
      RETURN
      END
```

```
      SUBROUTINE PIPE(FF,VX,EV,H,GLIM)
C     H = ENTRANCE MACH NUMBER FOR CHOKED FLOW   EMX=1
C     EM =MACH NUMBER AT ENTRANCE OF PIPE.
C     EMX=MACH NUMBER AT EXIT OF PIPE
C     G  = RATIO OF SPEC. HEAT
C     FF = PIPE FRICTION FACTOR
C     EP = PRESSURE AT ENTRANCE
C     ER = DENSITY AT ENTRANCE
C     P1 =PRESSURE AT EXIT
C     VX = VELOCITY OF SOUND AT EXIT
C     EV = VELOCITY OF SOUND AT ENTRANCE
C     R1 = DENSITY AT EXIT
      DIMENSION D(41),X(41)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX
      IJ=1
      G1=(G+1.)/2.
      G2=(G-1.)/2.
      B=EM**2
      A2=FF-(1./B)-G1*ALOG(ABS(B/(1.+G2*B)))
      A1=-1.+G1*ALOG(G1)
      IF(A2-A1)8,13,13
   13 EMX=1.
      IJ=2
    8 D(1)=1.
      GO TO 10
   35 D(1)=X1
   10 DO 34 I=1,40
      GO TO(1,2),IJ
    1 Z1=ALOG(B*(1.+G2*D(I))/(D(I)*(1.+G2*B)))
      X1=1./(1./B-FF+G1*Z1)
      X(I)=X1-D(I)
      EMX=SQRT(ABS(D(I)))
      F=EMX**2
      GO TO 11
    2 Z1=ALOG(ABS(D(I)*G1/(1.+G2*D(I))))
      X1=(1./(1.+FF-G1*Z1))
      X(I)=X1-D(I)
      H=SQRT(ABS(D(I)))
   11 IF(I-1)15,15,7
   15 D(2)=.1
      GO TO 34
    7 D(I+1)=D(I)-X(I)*(D(I)-D(I-1))/(X(I)-X(I-1))
      IF(ABS(X(I))-1.0E-08)6,6,34
   34 CONTINUE
      GO TO 35
```

```
  6 GO TO(21,14),IJ
21   IJ=2
     GO TO 8
14   IF(EMX-1.)17,3,3
17   XX=1.+G2*F
     XY=1.+G2*B
     P1(1)=EP*SQRT(ABS(B*XY/(F*XX)))
     VX=EV*SQRT(ABS(XY/XX))
     R1(1)=ER*SQRT(ABS(B*XX/(F*XY)))
     Z1=H+H
     X1=1.+G2*Z1
     X2=1.+G2*B
     GLIM=G*EP*SQRT(Z1*X1/(B*X2))*H*A(1)/(EV*SQRT(X2/X1))
     GO TO 4
  3  B1=H*H
     XX=1.+G2*B1
     XY=1.+G2*B
     EP=EP/SQRT(B1*XX/(B*XY))
     ER=ER*SQRT(B*XX/(B1*XY))
     EV=EV*SQRT(XY/XX)
     P1(1)=EP*H*SQRT(XX/G1)
     VX=EV*SQRT(XX/G1)
     R1(1)=ER*H*SQRT(G1/XX)
     EM=H
     GLIM=G*EP*H*A(1)/EV
  4  RETURN
     END
```

```
      SUBROUTINE ITER(NU,NS,PM,DE,DEM,PM1,MSUB,MSON)
      DIMENSION DPX(40),AMA(40),DPE(40),AMB(40)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX
      MSUB=1
      MSON=1
      IF(EMX-1.)1,3,3
    1 NU=NU+1
      ANU=NU
      DPX(NU)=DE
      AMA(NU)=PM
      IF(NU-2)6,2,2
    2 AA=ABS(AMA(NU)-AMA(NU-1))
      IF(AA-1.0E-07*P1(2))9,12,12
   12 PM1=AMA(NU)-DPX(NU)*(AMA(NU)-AMA(NU-1))/(DPX(NU)-DPX(NU-1))
      GO TO 5
    9 MSUB=0
      IF(ABS(DE)-(1.0E-07*P1(2)))8,8,15
   15 IF(ABS(DEM)-2.0E-06)16,16,8
   16 MSUB=1
      EMX=1.0
    3 NS=NS+1
      ANS=NS
      DPE(NS)=DEM
      AMB(NS)=PM
      IF(NS-2)8,4,4
    4 AB=ABS(AMB(NS)-AMB(NS-1))
      IF(AB-1.0E-07*P1(2))13,14,14
   13 MSON=0
      GO TO 8
   14 PM1=AMB(NS)-DPE(NS)*(AMB(NS)-AMB(NS-1))/(DPE(NS)-DPE(NS-1))
      IF(PM1-AMB(NS))10,11,10
   11 PM1=AMB(NS)*(ANS+0.010)/ANS
   10 IF(ABS(DEM)-1.0E-06)7,7,5
    7 IF(DE+1.0E-07)1,5,5
    6 PM1=(PM+P2(2))/2.0
      GO TO 5
    8 PM1=.9995*PM
    5 RETURN
      END
```

```
      SUBROUTINE ENTR(K,SV)
C     AM1= MASS FLOW FROM UPPER COMPARTMENT
C     P1 = MEAN PRESS. OF UPPER COMPARTMENT =TOTAL PRESSURE
C     R1 = MEAN DENSITY OF UPPER COMP. ; TOTAL DENSITY
C     G  = SPEC. HEAT RATIO
C     A  = PIPE AREA
C     EP = PRESSURE IN PIPE ENTRANCE (MEAN)
C     EM = MACH NUMB. AT ENTR. OF PIPE
C     ER = DENSITY AT ENTR. OF PIPE
C     SV = VELOCITY OF SOUND IN THE ENTRANCE
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      D=(2./(G+1.))**(G/(G-1.))
      K1=K+1
      DO 2 I=1,10
      IF(AL(K1))11,11,12
   11 AL1(K1)=.0
      GO TO 13
   12 Z1=2.0*PA1(K1)/(P2(K1)+P1(K1))
      TM=(P2(K1)+P1(K1))/2.0
      CALL LEAK(Z1,TM,R1(K1),AL(K1),D,HR(K1),CCF(K1),AL1(K1))
   13 S1=R2(K1)*VO(K1)
      AM1(K)=AM1(K1+1)-AL1(K1)+S1*(1.-((P1(K1)/P2(K1))**(1./G)))
      R1(K1)=R2(K1)*(1.+(AM1(K1+1)-AL1(K1)-AM1(K))/S1)
      A0=((AM1(K)/A(K))**2)/(G*P1(K1)*R1(K1))
      B=.73958*G
      A1=(2.*A0*B-1.)/(G-1.-2.*B*B*A0)
      A2=(2.*A0)/(G-1.-2.*B*B*A0)
      FMP=A1+SQRT(ABS(A2+A1*A1))
      RPT=(1.+(G-1.)*FMP/2.)**(G/(1.-G))
      PT2=P1(K1)/(RPT*(1.0+B*FMP))
      PTR=1.0/(RPT*(1.0+B*FMP))
      EP=RPT*PT2
      ER=R1(K1)*PT2*((1.+(G-1.)*FMP/2.)**(1./(1.-G)))/P1(K1)
      EM=SQRT(ABS(FMP))
      SV=SQRT(ABS(G*EP/ER))
      IF(PTR-1.)6,6,1
    6 IF(A0-(1.0+G)/(2.0*((1.+B)**2)))3,3,1
    1 P1(K1)=(P1(K1)+P2(K1))/2.0
    2 CONTINUE
    3 IF(MPRNT-2)4,4,5
    5 WRITE(61,100)PTR,FMP,SV,EP,ER,A1,A0,P1(K1)
    4 RETURN
  100 FORMAT(8E15.8)
      END
```

```
      SUBROUTINE FRICT(Y,AC,T,FR,A,R)
      DIMENSION F(41)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G
C    Y  =MASS FLOW
C    AC =PIPE AREA
C    T  =GAS TEMP.
C    TC =CRIT. TEMP.
C    VC =CRIT. VISC.
C    FR =FRICTION FACTOR
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      CALL VISC(T,TC,VC,A)
      D=SQRT (4.*AC/3.1416)
      R=ABS (Y*D*9.8/(AC*A))
      IF(R)4,4,5
    4 FR=.0
      GO TO 3
    5 F(1)=.05
      DO 1 I=1,40
      F(I+1)=1./((.86858*ALOG(ABS (R*SQRT (F(I))))-.8)**2)
      FR=F(I+1)
      IF(I-1)1,1,2
    2 IF(ABS (F(I+1)-F(I))-1.0E-07)3,1,1
    1 CONTINUE
      IF(.86858*ALOG(ABS(R*SQRT(FR)))-.8)6,6,7
   6  FR=10.0
      GO TO 3
   7  WRITE(61,100)
    3 RETURN
 100  FORMAT(3X,15HFRICT TOLERANCE)
      END
```

```
      SUBROUTINE TRAJ(ALT,FM,HG,PA,TA,RA,CA)
C     PA=AMBIENT PRESSURE
C     TA=TEMPERATURE
C     RA=DENSITY
C     CA=SPEED OF SOUND
C     PA1(N)=PRESSURE OUTSIDE PIPE AND LEAKS AT TIME T1
      DIMENSION W1(40)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      CALL INTER(1,NH,1,1,TT,HT,T1,ALT)
      CALL INTER(1,NH,1,1,TT,FMT,T1,FM)
      DO 1 I=1,N
      DO 2 J=1,NP
    2 W1(J)=CPF(J,I)
      CALL INTER(1,NP,1,1,FMF,W1,FM,PA1(I))
    1 CONTINUE
      HG=ALT*6.378178E+06/(6.378178E+06+ALT)
      CALL ATMOS(HG,PA,TA,RA,CA)
      DO 3 I=1,N
    3 PA1(I)=PA+RA*PA1(I)*((FM*CA)**2)/2.0
      RETURN
      END
```

```
      PROGRAM VENT
C   VENTING THROUGH A PIPE, HEAT ADDITION TO PIPE
C    ************************************************************
C    NH = NUMBER OF CARDS FOR TRAJECTORY DATA
C    N  =NUMBER OF COMPARTMENTS,ALSO NUMBER OF LEAKS
C        ONE LEAK PER COMPARTMENT
C        PIPE IS CONSIDERED AS COMPARTMENT NO. 1
C   HT = ALTITUDE AT TIME TT IN METERS
C   FMT = MACH NUMBER AT TIME TT
C    A  = ORIFICE AREA BETWEEN N COMP.IN METERS
C    VO = VOLUME OF COMPARTMENTS IN METERS3
C    CPF= PRESSURE COEFFICIENT AT MACH N.
C    A  = AREA OF NV ORIFICES OF COMP. 1 IN METERS
C    P  = PRESSURE IN COMPARTMENT IN (KG/M2)
C    CM1= COMPARTMENT MACH NUMBER
C    AL1= LEAK MASS FLOW
C    R  = DENSITY OF N COMP. IN KG.SEC./M4
C    AL =LEAK AREA OF COMP.N IN METERS
C    HR =HYDRAULIC RADIUS OR MEAN RADIUS IN METER
C    TC = CRITICAL TEMPERATURE OF GAS
C    GC = GAS CONSTANT OF GAS (M/DEG.K)
C    PC = CRITICAL PRESSURE OF GAS IN ATM(KG/CM2)
C    OM = MOL WEIGHT OF PARTICULAR GAS
C    CCF= RADIAL CLEARANCE OVER FLOW LENGTH
C    WI =ADDITIONAL LOSS COEFFICIENT FOR PIPE, IF NONE ENTER 0.
C        IN PROPER FORMAT
C        NUMBER 1 COMP. IS THE COMP. THAT VENTS INTO THE ATMOSPHERE
C    QT = HEAT ADDITION ALONG TRAJECTORY
C    Q  = TOTAL HEAT ADDITION TO THE PIPE AT TIME T1
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5),QT(40),Q,AM3(5),P3(5)
      DO 5 I=1,5
      AM1(I)=0.0
    5 AM2(I)=1.0E-07
      CALL INPUT
      D=(2./(G+1.))**(G/(G-1.))
      DO 13 I=NO,NT
      I1=I-1
      T1=I1
      T2=T1-1.0
      CALL TRAJ
      IF(I-1)4,11,4
    4 DO 15 L=1,20
      IF(L-1)6,6,7
    6 DO 8 K=1,N
```

```
      P1(K)=2.0*P2(K)-P3(K)
    8 AM1(K)=2.0*AM2(K)-AM3(K)
    7 DO 20 K=2,N
      IF(K-2)1,1,2
    1 CONTINUE
      CALL PARTB(K,L,I)
      GO TO 23
    2 CONTINUE
      CALL PARTA(I,K,L,D)
   23 WH(L,K)=P1(K-1)
   20 WM(L,K)=AM1(K)
      IF(L-1)10,10,9
    9 DO 3 K=1,N
      DEL=(WM(L,K)-WM(L-1,K))/WM(L,K)
      IF(MPRNT)18,18,19
   19 WRITE(61,91)K,WM(L,K),WM(L-1,K),DEL
   18 IF(ABS((WM(L,K)-WM(L-1,K))/WM(L,K))-5.0E-03)3,3,10
    3 CONTINUE
      GO TO 12
   10 IF(L-3)15,16,16
   16 DO 17 K=3,N
   17 P1(K-1)=WH(L,K)-(WM(L,K)-WM(L-1,K))*(WH(L,K)-WH(L-1,K))/(WM(L,K)-
     12.0*WM(L-1,K)+WM(L-2,K))
      DO 24 K=1,N
   24 AM1(K)=(WM(L,K)+WM(L-1,K))/2.0
      IF(MPRNT)15,15,22
   22 WRITE(61,90)L,L,AM1(1),AM1(2),AM1(3),AM1(4),P1(2),P1(3)
   15 CONTINUE
      GO TO 12
   11 EM=0.0
      EMX=0.0
      DO 14 IP=1,N
      CM1(IP)=+0.0
   14 AL1(IP)=+0.0
   12 CALL OUTPUT(I,L)
      WRITE(61,555)
   13 CONTINUE
      STOP
  555 FORMAT(1H0)
   90 FORMAT(2I4,8E15.8)
   91 FORMAT(I4,5E15.8)
      END
```

```
      SUBROUTINE PARTB(K1,L1,I)
      DIMENSION PM(41)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),P2(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5),QT(40),Q
      K=K1-1
      NU=0
      NS=0
      DO 1 L=1,40
      IF(MPRNT-1)2,2,10
   10 WRITE(61,555)
      WRITE(61,100)L,K
    2 IF(L-2)6,3,5
    6 PM(L)=P1(K1)
      GO TO 5
    3 PM(L)=.998*PM(L-1)
    5 P1(K1)=PM(L)
      TO=P1(K1)/(R1(K1)*GC*9.8)
      CALL ENTR(K,SV)
      PM(L)=P1(K1)
      AM1(K1)=AM1(K)
      HPO=EM
      TL=TO/(1.+(G-1.)*EM *EM /2.)
      CALL FRICT(AM1(K ),A(K),TL,FR,VIS,RN)
      FF=G*(FR*PL(K)/(2.*HR(K))+W)
      CALL PIPE(FF,CE,SV,HP,TO,TX)
      IF(MPRNT-1)11,11,12
   12 WRITE(61,100)L,K,L1,P1(K),EP,EM,EMX,PA1(K),AM1(K),AM1(K1),HP
      WRITE(61,100)L,K1,L1,P1(K1),R1(K1),P2(K1),P2(K)
   11 DE=P1(K)-PA1(K)
      DEM=HPO-HP
      CALL ITER(NU,NS,PM(L),DE,DEM,PM(L+1),MSB,MSO)
      IF(MPRNT-1)13,13,14
   14 WRITE(61,101)L,K,AM1(K1),PM(L),PM(L+1),DE,DEM
   13 IF(EMX-1.)17,7,7
   17 IF(MSB)4,8,4
    4 IF(ABS(DE)-(1.0E-07*P1(K1)))8,8,1
    7 IF(DE+(2.0E-07*P1(K1)))1,9,9
    9 IF(ABS(DEM)-1.0E-04*EM)8,8,18
   18 IF(MSO)1,8,1
    1 CONTINUE
    8 WM(L1,K)=AM1(K)
      WM(L1,K1)=AM1(K1)
```

```
      CM1(K1)=EM
      CM1(K)=EMX
      IF(MPRNT)15,15,16
 16   WRITE(61,102)L1,L,MSB,MSO,AM1(K),P1(K1),P2(K1)
 15   RETURN
101   FORMAT(2I4,6E15.8)
100   FORMAT(3I4,8E15.8)
102   FORMAT(4I4,8E15.8)
555   FORMAT(1H0)
      END
```

```
      SUBROUTINE INPUT
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5),QT(40),Q,AM3(5),P3(5)
      READ(60,100)NH,NP,MPRNT
      READ(60,101)OM,TC,PC,GC,G
      VC=7.7*SQRT (OM)*(TC**(-1./6.))*(PC**(2./3.))*1.0E-07
      WRITE(61,126)GC,OM,TC,PC
      WRITE(61,128)VC
      DO 1 I=1,NH
1     READ(60,101) TT(I),HT(I),FMT(I),QT(I)
      DO 2 J=1,NP
2     READ(60,101)FMF(J),CPF(J,1),CPF(J,2),CPF(J,3),CPF(J,4)
      READ(60,100)NO,NT,N
      WRITE(61,130)
      READ(60,101) A(1),HR(1),PL(1),R1(1),P1(1)
      WRITE(61,124)A(1),HR(1),PL(1)
      WRITE(61,108)
      DO 5 I=2,N
      READ(60,101) A(I),VO(I),R1(I),P1(I),AM1(I)
      READ(60,101) AL(I),HR(I),CCF(I),AL1(I),AM2(I)
      READ(60,101) P2(I)
      R2(I)=R1(I)
      P3(I)=P2(I)
      AM3(I)=AM2(I)
      P2(I)=P1(I)
      AL2(I)=AL1(I)
      AM2(I)=AM1(I)
      WRITE(61,125)I, A(I),VO(I),R1(I),P1(I)
5     WRITE(61,127) AL(I),HR(I),CCF(I)
      AM3(1)=AM2(2)
      AM2(1)=AM1(2)
      EP=P1(2)
      ER=R1(2)
      READ(60,101)W
      WRITE(61,108)
      RETURN
  124 FORMAT(10H PIPE AREA,E15.8,10H HYDR.RAD.,E15.8,8H PIPE L.,E15.8)
  130 FORMAT(13H PIPE SECTION)
  101 FORMAT(5E15.8)
  100 FORMAT(9I3)
  128 FORMAT(15H CRIT.VISCOSITY,E13.6,6H KG/MS)
  126 FORMAT(11H GAS CONST.,E13.6,5H M/OK,7H MOL W.,E13.6,9H CR.TEMP.,E1
     13.6,3H OK,9H CR.PRES.,E13.6,4H ATM)
  127 FORMAT(22H                LEAK AREA,E15.8,3H M2,10H HYDR.RAD.E13.6,2H
     1 M,16H RAD.CLEAR/FL.L.,E13.6)
  125 FORMAT(9H COMP.NR.,I3,10H ORIF.AREA,E15.8,3H M2,5H VOL.,E15.8,3H M
     13,8H DENSITY,E15.8,8H KGS2/M4,7H PRES. E15.8,6H KG/M2)
 108  FORMAT(1H0)
      END
```

```
      SUBROUTINE TRAJ
C     PA=AMBIENT PRESSURE
C     TA=TEMPERATURE
C     RA=DENSITY
C     CA=SPEED OF SOUND
C     PA1(N)=PRESSURE OUTSIDE PIPE AND LEAKS AT TIME T1
      DIMENSION W1(40)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5),QT(40),Q
      CALL INTER(1,NH,1,1,TT,HT,T1,ALT)
      CALL INTER(1,NH,1,1,TT,FMT,T1,FM)
      CALL INTER(1,NH,1,1,TT,QT,T1,Q)
      DO 1 I=1,N
      DO 2 J=1,NP
    2 W1(J)=CPF(J,I)
      CALL INTER(1,NP,1,1,FMF,W1,FM,PA1(I))
    1 CONTINUE
      HG=ALT*6.378178E+06/(6.378178E+06+ALT)
      CALL ATMOS(HG,PA,TA,RA,CA)
      DO 3 I=1,N
    3 PA1(I)=PA+RA*PA1(I)*((FM*CA)**2)/2.0
      RETURN
      END
```

```
      SUBROUTINE PIPE(FR,CX,CE,H,TO,TX)
C   SUBROUTINE FOR CALCULATING PIPE FLOW WITH CONST. HEAT ADDITION
C   Q  = TOTAL HEAT INPUT
C   PL = PIPE LENGTH
C   TO     = TOTAL TEMPERATURE AT ENTRANCE OF PIPE
C   FR = FRICTION TERM (4F)/D
C   G = SPEC. HEAT RATIO
C   GC = GAS CONSTANT
C   EN = ENTRANCE MACH NUMBER
C   PE = PRESSURE AT ENTRANCE OF PIPE
C   EX = EXIT MACH NUMBER
C   PX = PRESSURE AT EXIT
C   RE = DENSITY AT ENTRANCE
C   RX = DENSITY AT EXIT
C   CE  SPEED OF SOUND AT ENTRANCE
C   CX  SPEED OF SOUND AT EXIT
C    H  = ENTRANCE MACH NUMBER FOR CHOKED FLOW, EMX=1
C    PL  = PIPE LENGTH
      DIMENSION T(21),A(21),FX(51),FM(51),TM(51)
      COMMON MPRNT
      COMMON NH,NZ,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FT(40),FMF(40),W,NO
      COMMON CF(40,5),AA(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),TI,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FMM
      COMMON WM(40,5),QT(40),Q
      N=6
      M=20
      IJ=M+1
      A1=M*N
      DX=1./A1
      CP=G*GC/(G-1.)
      DT=Q/CP
      JE=N+1
      B=EM*EM
      G1=(G-1.)/2.
      G2=G-1.
      G3=(G+1.)/2.
      T(1)=TO
      TM(1)=TO
      A(1)=EM**2
      FX(1)=0.
      FM(1)=A(1)
      CALL PIPEA(JE,M,N,ME,T,TM,A,FX,FM,DX,DT,SLP,AM,TO1,FR)
      TERM=ME
      XO=TERM/20.
      TX=T(ME+1)
      EMX=FM(ME+1)
      EX=EMX
      MA=0
```

```
      IF(MPRNT-2)3,3,5
    5 WRITE(61,90)JE,M,N,ME,MA,SLP,X0,TX,EMX,EX,T01,DT
   90 FORMAT(5I3,8E15.8)
    3 IF(SLP-.0020)65,65,66
   66 TERM=ME-1
      X0=TERM/20.
      AM=A(ME)
      T01=T(ME)
      TM(ME)=T(ME)
   65 IF(X0-1.)7,9,9
    7 N=20
      A1=N
      DM2=(1.-AM)/20.
      DM1=DM2/A1
      A(1)=AM
      T(1)=T01
      CALL PIPEB(JE,M,N,MA,ME,T,TM,A,FX,FM,DT,DM1,DM2,TX,FR,X0)
    9 CONTINUE
      M1=ME+MA
      KK=1
      TM(M1)=TX
      IF(EMX-1.)1,2,2
    1 FX(M1)=1.0
      FM(M1)=EMX
      EXA=SQRT(EMX)
      H=EM*SQRT(1./EXA)
      GO TO 4
    2 EMX=1.0
      EXA=1.0
      H=EM*SQRT(FX(M1))
    4 DST=TX*(1.+G1*EM*EM)/(T0*(1.+G1*EMX))
      P1(1)=EP*EM*SQRT(DST)/EXA
      R1(1)=ER*P1(1)/(DST*EP)
      CX=SQRT(G*P1(1)/R1(1))
      EMX=SQRT(EMX)
      IF(MPRNT-2)6,6,8
    8 WRITE(61,91)M1,KK,EMX,FX(M1),TM(M1),EXA,H,P1(1),R1(1)
   91 FORMAT(2I3,8E15.8)
      WRITE(61,91)M1,KK,TX,T0,DT,PA1(1)
    6 RETURN
      END
```

```
      SUBROUTINE PIPEA(JE,M,N,ME,T,TM,A,FX,FM,DX,DT,SLP,AM,T01,FR)
      DIMENSION T(21),A(21),FX(51),FM(51),TM(51)
      DIMENSION Y(91),S(91),T1(91),S1(91),BM(91)
      COMMON MPRNT
      COMMON NH,NZ,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FT(40),FMF(40),W,NO
      COMMON CF(40,5),AA(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FMM
      COMMON WM(40,5),QT(40),Q
      TOL=1.0E-06
      IJ=M+1
      G1=(G-1.)/2.
      G2=G-1.
      G3=(G+1.)/2.
      DO 1 I=1,M
   48 A1=A(I)*(1.+G*A(I))*(1.+G1*A(I))*DT/(T(I)*(1.-A(I)))
      A2=G*(A(I)**2)*(1.+G1*A(I))*FR/(1.0*(1.-A(I)))
      DO 42 J=1,JE
      X=J-1
      Y(J)=(A1+A2)*X*DX+A(I)
      S(J)=Y(J)
   42 T1(J)=T(I)+DT*DX*X
      T(I+1)=T1(JE)
      TM(I+1)=T(I+1)
   46 DO 2 J=1,JE
      A1=S(J)*(1.+G*S(J))*(1.+G1*S(J))*DT/(1.0*T1(J)*(1.-S(J)))
      A2=G*(S(J)**2)*(1.+G1*S(J))*FR/(1.0*(1.-S(J)))
    2 S1(J)=(A1+A2)
      CALL INT(JE,DX,S1,BM)
      DO 3 J=1,JE
    3 S(J)=A(I)+BM(J)
      IF(S(JE))70,71,71
   71 IF(S(JE)-.95)72,72,70
   70 NL=JE+1
      GO TO 49
   72 DO 43 J=1,JE
      IF(MPRNT-5)4,4,5
    5 SSJ=SQRT(S(J))
      SYJ=SQRT(Y(J))
      WRITE(61,90)I,J,JE,SSJ,SYJ,TOL
    4 IF(ABSF(SQRTF(S(J))-SQRTF(Y(J)))-TOL)43,43,44
   43 CONTINUE
      NL=JE+1
      GO TO 20
   44 NL=J
   20 DO 35 J=1,JE
   35 Y(J)=S(J)
      SLP=S1(JE)*DX
      IF(I-IJ)36,34,34
```

```
 36 DO 45 J=1,JE
    IF(S1(J)*DX-.0020)45,45,33
 45 CONTINUE
    GO TO 34
33  IF(JE-60)51,51,49
51  N=N+4
    JE=N+1
    A1=M*N
    DX=1./A1
    GO TO 48
49  IJ=I
34  A(I+1)=S(JE)
    ME=I
    AM=A(I+1)
    T01=T(I+1)
    IF(NL-(JE+1))46,37,37
37  A1=I
    FX(I+1)=A1/20.
    FM(I+1)=A(I+1)
    TM(I+1)=T(I+1)
    IF(I-IJ)1,54,54
  1 CONTINUE
54  RETURN
90  FORMAT(3I4,8E15.8)
    END
```

```
      SUBROUTINE PIPEB(JE,M,N,MA,ME,T,TM,A,FX,FM,DT,DM1,DM2,TX,FR,X0)
      DIMENSION Y(91),T1(91),Z(91),S(91),S1(91),BM(91),R(91),P(51),C(51)
      DIMENSION T(21),A(21),FX(51),FM(51),TM(51)
      COMMON MPRNT
      COMMON NH,NZ,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FT(40),FMF(40),W,NO
      COMMON CF(40,5),AA(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FMM
      COMMON WM(40,5),QT(40),Q
      TOL=1.0E-06
      G1=(G-1.)/2.
      G2=G-1.
      G3=(G+1,)/2.
      C(1)=X0
      DO 4 I=1,M
53    A1=(1.-A(I))/(A(I)*(1.+G1*A(I)))
      A2=1./((1.+G*A(I))*DT/T(I)+G*A(I)*FR)
      SLP=A1*A2*DM1
      IF(N-60)62,62,61
   62 IF(N-6)60,63,63
   63 IF(A1*A2*DM1-.0025)39,38,40
   39 IF(A1*A2*DM1-.0015)41,41,38
   41 N=N-2
      JE=N+1
      IF(N-6)60,52,52
   60 N=6
      JE=7
      GO TO 38
   40 N=N+4
      JE=N+1
      IF(N-60)52,61,61
   61 N=60
      JE=61
      GO TO 38
52    A1=N
      DM1=DM2/A1
      GO TO 53
38    DO 5 J=1,JE
      X=J-1
      IF(J-1)18,18,19
   18 Y(J)=C(I)
      T1(J)=T(I)
      Z(J)=Y(J)
      GO TO 5
   19 Y(J)=Y(J-1)+A1*A2*DM1
      T1(J)=T1(J-1)+DT*(Y(J)-Y(J-1))
      Z(J)=Y(J)
    5 S(J)=A(I)+DM1*X
      A(I+1)=S(JE)
```

```
   14 DO 8 J=1,JE
      A1=(1.-S(J))/(S(J)*(1.+G1*S(J)))
      A2=1./((1.+G*S(J))*DT/T1(J)+G*S(J)*FR)
    8 S1(J)=(A1*A2)
      CALL INT(JE,DM1,S1,BM)
      DO 10 J=1,JE
      Z(J)=C(I)+BM(J)
      IF(J-1)16,16,17
   16 T1(J)=T(I)
      GO TO 10
   17 T1(J)=T1(J-1)+DT*(Z(J)-Z(J-1))
   10 CONTINUE
      DO 11 J=1,JE
      IF(MPRNT-5)1,1,2
    2 WRITE(61,90)I,J,JE,Z(J),Y(J),TOL
    1 IF(ABSF(Z(J)-Y(J))-TOL)11,11,12
   11 CONTINUE
      NL=JE+1
      GO TO 21
   12 NL=J
   21 DO 13 J=1,JE
   13 Y(J)=Z(J)
      C(I+1)=Z(JE)
      T(I+1)=T1(JE)
      IF(NL-(JE+1))14,6,6
    6 MA=I
      M2=ME+I
      FX(M2)=C(I+1)
      FM(M2)=A(I+1)
      TM(M2)=T(I+1)
      IF(C(I+1)-1.)22,26,26
   26 R(1)=1.
      CALL INTER(1,JE,1,1,Z,S,R,P)
      EMX=P(1)
      CALL INTER(1,JE,1,1,Z,T1,R,P)
      TX=P(1)
      GO TO 27
   22 IF(A(I+1)-1.)15,28,28
   28 EMX=1.0
      TX=T(I+1)
      GO TO 27
   15 IF(I-M)4,28,28
    4 CONTINUE
   27 RETURN
   90 FORMAT(3I4,8E15.8)
      END
```

```
      SUBROUTINE ITER(NU,NS,PM,DE,DEM,PM1,MSUB,MSON)
      DIMENSION DPX(40),AMA(40),DPE(40),AMB(40)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX
      MSUB=1
      MSON=1
      IF(EMX-1.)1,3,3
    1 NU=NU+1
      ANU=NU
      DPX(NU)=DE
      AMA(NU)=PM
      IF(NU-2)6,2,2
    2 AA=ABS(AMA(NU)-AMA(NU-1))
      IF(AA-1.0E-07)9,12,12
   12 PM1=AMA(NU)-DPX(NU)*(AMA(NU)-AMA(NU-1))/(DPX(NU)-DPX(NU-1))
      GO TO 5
    9 MSUB=0
      IF(ABS(DE)-(1.0E-07*P1(2)))8,8,15
   15 IF(ABS(DEM)-2.0E-06)16,16,8
   16 MSUB=1
      EMX=1.0
    3 NS=NS+1
      ANS=NS
      DPE(NS)=DEM
      AMB(NS)=PM
      IF(NS-2)8,4,4
    4 AB=ABS(AMB(NS)-AMB(NS-1))
      IF(AB-1.0E-07)13,14,14
   13 MSON=0
      GO TO 8
   14 PM1=AMB(NS)-DPE(NS)*(AMB(NS)-AMB(NS-1))/(DPE(NS)-DPE(NS-1))
      IF(PM1-AMB(NS))10,11,10
   11 PM1=AMB(NS)*(ANS+0.010)/ANS
   10 IF(ABS(DEM)-1.0E-04*EM)7,7,5
    7 IF(DE+1.0E-07)1,5,5
    6 PM1=(PM+P2(2))/2.0
      GO TO 5
    8 PM1=.9995*PM
    5 RETURN
      END
```

```
      SUBROUTINE OUTPUT(INT,L)
      COMMON MPRNT
      COMMON NH,N,NT,NP,GC,OM,TC,PC,G,TT(40),HT(40),FMT(40),FMF(40),W,NO
      COMMON CPF(40,5),A(5),HR(5),PL(1),VO(5),AL(5),CCF(5),PA1(5),PZ(1)
      COMMON P1(5),R1(5),AM1(5),T1,CM1(5),AL1(5)
      COMMON P2(5),R2(5),AM2(5),T2,CM2(5),AL2(5)
      COMMON WH(40,5),TP(40),OP(40),EP,ER,EM,EMX,PA,TA,RA,CA,ALT,FM
      COMMON WM(40,5),QT(40),Q,AM3(5),P3(5)
      WRITE(61,50)T1,L
 50   FORMAT(11H TIME       E12.5,11H NR.ITER.  I3)
      WRITE(61,60)ALT,FM,PA,TA,RA
 60   FORMAT(11H ALTITUDE  E12.5,11H MACH NR.  E12.5,11H AMB.PR.   E12.5
     1,11H AMB.TEMP  E12.5,11H AMB.DENS  E12.5)
      WRITE(61,61)EP,ER,EM,AM1(1)
 61   FORMAT(34H COMPARTM.     PIPE       ENTR.PR.  E12.5,11H ENTR.DENS.E1
     12.5,11H ENTR.M.NR.E12.5,11H ENTR.M.FL.E12.5)
      WRITE(61,62)P1(1),R1(1),EMX,AM1(1)
 62   FORMAT(34H                          EXIT PR.  E12.5,11H EXIT.DENS E1
     12.5,11H EXIT M.NR.E12.5,11H EXIT M.FL.E12.5)
      WRITE(61,65)Q
 65   FORMAT(34H                          HEAT AD.  E12.5)
      DO 1 I=2,N
      DPN=P1(I)-PA
      WRITE(61,63)I,P1(I),R1(I),CM1(I),AM1(I)
  1   WRITE(61,64)PA1(I),AL1(I),DPN
 63   FORMAT(12H COMPARTM.  I3,19H           PRESSURE  E12.5,11H DENSITY
     1 E12.5,11H MACH NR   E12.5,11H MASS FLOW E12.5)
 64   FORMAT(34H                          LEAK PR.  E12.5,11H LEAK M.FL.E1
     12.5,11H PRES.DIFF.E12.5)
      IF(INT-NT)4,6,6
  6   NO1=NT+1
  9   NT1=201
      WRITE(62,80)NO1,NT1,N
      WRITE(62,81)A(1),HR(1),PL(1),R1(1),P1(1)
      DO 2 I=2,N
      WRITE(62,81)A(I),VO(I),R1(I),P1(I),AM1(I)
      WRITE(62,81)AL(I),HR(I),CCF(I),AL1(I),AM2(I)
  2   WRITE(62,81)P2(I)
      WRITE(62,81)W
      GO TO 10
  4   GO TO (8,10),SSWTCHF(1)
  8   NO1=INT+1
      GO TO 9
 10   DO 3 I=1,N
      AM3(I)=AM2(I)
```

```
      P3(I)=P2(I)
      AM2(I)=AM1(I)
      IF(INT-1)5,7,5
    7 AM2(I)=AM1(I)+1.0E-06
    5 P2(I)=P1(I)
      R2(I)=R1(I)
      CM2(I)=CM1(I)
    3 AL2(I)=AL1(I)
      AM2(N+1)=0.0
      AM1(N+1)=0.0
      EP=.99*EP
      RETURN
   80 FORMAT(9I3)
   81 FORMAT(5E15.8)
      END
```

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COMPARTMENT VENTING AND PIPE FLOW WITH HEAT ADDITION

by H. G. Struck and John A. Harkins

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